Students with low reading abilities and word problems in mathematics

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In this paper we looked into whether text in word problems is an extra challenge for students in 5th and 6th grade with low reading abilities. We analysed data from four tasks, which were part of a larger survey sample and compared students with low and satisfactory reading abilities. Our findings indicate that students reading abilities affects their mathematical ability.

Keywords: Word problem, low and satisfactory reading ability, mathematical ability

Introduction

Word problems (WPs) in mathematics is not a recent notion. Some of the earliest example of human writing take the form of WPs (Swetz, 2009). “The term *word problem* is used to refer to any math exercise where significant background information on the problem is presented as text rather than in mathematical notation” (Boonen, Van der Schoot, Van Wesel, De Vries, & Jolles, 2013, p. 271).

Since most students face mathematics-related problems in written form in an out-of-school setting, it is natural that they should be taught and evaluated on their ability to solve WP (Helwig, Rozek-Tedesco, Tindal, Heath, & Almond, 1999). When students solve a WP, they first have to read the text and then solve the problem. Students draw on both mathematical competence and general reading strategies when they solve WPs (Nortvedt, 2013). There has been a discussion about whether or not the extensive use of WPs makes it harder for students with low reading abilities (LRA) to learn mathematics. The purpose of this paper is to see if students with LRA struggle more with WPs in mathematics than students with satisfactory reading abilities (SRA).

Theory

Students confronted with WPs in school “are engaged in a peculiar kind of activity wherein they typically solve these problems in a stereotyped and artificial way without relating them to any real-life situation” (Verschaffel, Greer, & De Corte, 2000, p. 12). According to Boaler (2009), students should not be involved in solving WPs that is in a context that require them to engage partly in the real world while at the same time ignoring everything they know about the real world. Also Greer, Verschaffel, and Mukhopadhyay (2007) claims that student in mathematics learn to play what they call the “Word Problem Game” where one of the rules are “violations of your knowledge about the everyday world may be ignored” (Greer et al., 2007, p. 92). We agree with Boaler, that real world context is important. Still students will meet WPs, which are not in a real world context in school and in assessments, and therefore it is important to study if students are able to solve these WPs and finding explanations if this is not the case.

In the Norwegian curriculum for the common core subject of mathematics (LK06) reading are one of five basic skills. The basic skill *reading* defined as:

> Being able to read in Mathematics involves understanding and using symbolic language and forms of expression to create meaning from texts in day-to-day life, working life and from

> mathematics texts. (…) Reading in Mathematics involves sorting through information, analysing and evaluating form and content, and summarising information from different elements in the texts. (Utdanningsdirektoratet, 2013)

According to Nortvedt (2010) there is a strong positive correlation between numeracy and reading comprehension. She has studied how 8th grade students in Norway are responding on multistep arithmetic WPs on national test in numeracy and compared this result with students respond on the national test in reading comprehension. “Student’s reading levels explained 44 % of the variability in their scores on the multistep arithmetic word problem scale” (Nortvedt, 2010, p. 33).

Normally, a mathematical problem is defined as a task where no standard procedure are known to the students (English & Gainsburg, 2016). With this definition, not all WPs are a mathematical problem (Björkqvist, 2003). We can have WPs, which are/are not a problem solving task, and problem solving tasks, which are/are not WPs. There are several ways of defining level of difficulty in WPs. When solving a WP, the students first have to translate the text into an internally represented model of the problem. “The translation phase is related to linguistic and factual knowledge and requires the skill of number selection to solve word problem” (Kingsdorf & Krawec, 2014, p. 66). Students who create a visual-schematic representation of the situation to be solved seem to benefit from it, while a production of a pictorial representation is negative related to WP solving performance (Boonen et al., 2013). In a study of 128 6th grade students in the Netherlands, “the production of visual-schematic representations explains 21 % of the relation between spatial ability and word problem solving performance” (Boonen et al., 2013, p. 276). This can explain why some students can solve WPs and other cannot, but we do not have data to investigate this further. But this is still relevant, since many students can solve common arithmetical tasks and they show good text comprehension skills, and yet they fail to solve WPs correctly there must be other factors involved also (Daroczy, Wolska, Meurers, & Nuerk, 2015).

What makes WPs challenging for students? One can separate problems by looking at the number of steps required to solve them, one step is normally easier to solve than multistep WPs (G. Nortvedt, 2012). But since two-steps tasks are often more difficult linguistically also, we can not conclude that the reason for them being more difficult is arithmetical complexity (Daroczy et al., 2015). Another way of distinguishing easy problems from more complicated ones, is to look at the actual text in the WPs. For example, by counting the number of words, whether it is used difficult or easy language in the text, if there is unnecessary information or if there are words that point to a particular arithmetic operation (Kingsdorf and Krawec, 2014). A WP in a familiar context or in a context the students have a relationship to can also be crucial if students manage or fail to solve the problem. Daroczy et al. (2015) conclude that difficulties in solving WPs are influenced by the complexity of linguistic and numerical factors, and their interrelation. In this paper, we will discuss some of these factors.

Methods

In this study, we used data taken from a survey sample of mathematics from the project: *The Function of Special Education* (SPEED) (Haug, 2017), a joint research project between Hedmark University College and Volda University College. The mathematical survey in the SPEED-project had 40 multiple-choice items, with 7 possible answers including the possibility to answer “I do not know the answer”. Some of the wrong answers on these items are related to well known misconceptions. In this paper, we have chosen four tasks from the survey that relates to each other in form of multiplication. (See Figure 1) In the SPEED-project the students also responded to Carlsten reading test (Carlsten, 2002) as a measure of whether the student has LRA or SRA. In this test, a student is classified as having a functional literacy if he could read more than 80 words per minute with less than 15 % error on a reading test. On the 5th grade reading test there were 25 possible correct answers making more than 22 acceptable, for the 6th grade the test had 27 correct answer making more than 23 acceptable.

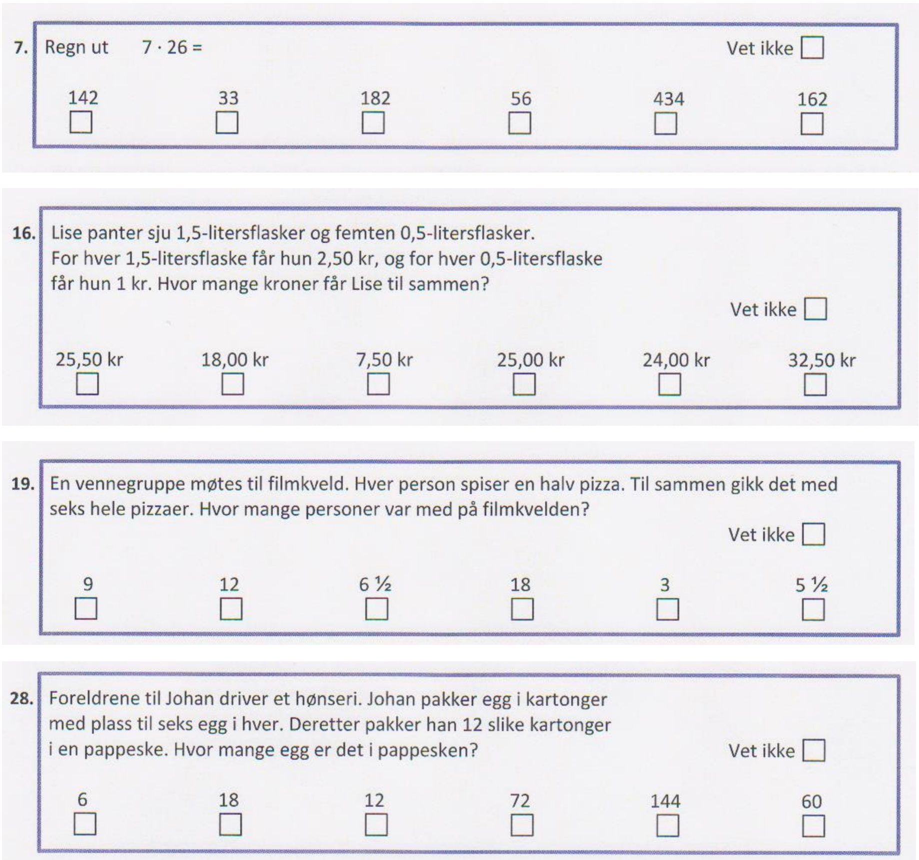
In our study, 593 students from 5th and 660 students from 6th grade participated. For each of the students their teacher provided an assessment of their academic achievement on a scale from 1 to 6, where 1 stands for very low skills and 6 for extraordinary skills in mathematics. Since we compared students according to their reading ability, we removed students rated at an academic achievement level equal to one or two in mathematics. In addition, we have also removed any students not assessed by their teacher. This left us with 475 students in 5th grade and 552 students in 6th grade.

In our study, 348 students in 5th and 475 students in 6th grade was classified as having SRA, leaving 127 and 77 with LRA in 5th and 6th grade, respectively. According to the curriculum in Norway, students after 4th grade are supposed to be able to do multiplication in practical situations. The focus is on the standard multiplication table, but also on using different methods for multiplication. After 7th grade students should be able to “reckon with positive (…) whole numbers, decimals,…” (Utdanningsdirektoratet, 2013). Since Norway have not divided the competencies between the 5th, 6th and 7th grade, it is difficult to tell whether the students have learned about multiplication with decimal numbers yet. We argue that if the students have not learned about it, then both the students with LRA and the students with SRA still have equal conditions.

Analysis and discussion

In our analysis we present result from task 7 first (see Figure 1). It is a standard multiplication task with single and two digit numbers, used here as a control task. Since this task has no context and only two words, the reading ability should not be a decisive factor. Both in 5th and 6th grade the students with SRA scored notable better than the students with LRA (Table 1). There is a significant difference[1] between the two groups for 6th grade (p=0.02), but not for the 5th grade (p=0.1).

[1] χ^2-test



7. Regn ut 7 · 26 = Vet ikke ☐

142 ☐ 33 ☐ 182 ☐ 56 ☐ 434 ☐ 162 ☐

16. Lise panter sju 1,5-litersflasker og femten 0,5-litersflasker.
For hver 1,5-litersflaske får hun 2,50 kr, og for hver 0,5-litersflaske
får hun 1 kr. Hvor mange kroner får Lise til sammen? Vet ikke ☐

25,50 kr ☐ 18,00 kr ☐ 7,50 kr ☐ 25,00 kr ☐ 24,00 kr ☐ 32,50 kr ☐

19. En vennegruppe møtes til filmkveld. Hver person spiser en halv pizza. Til sammen gikk det med
seks hele pizzaer. Hvor mange personer var med på filmkvelden? Vet ikke ☐

9 ☐ 12 ☐ 6 ½ ☐ 18 ☐ 3 ☐ 5 ½ ☐

28. Foreldrene til Johan driver et hønseri. Johan pakker egg i kartonger
med plass til seks egg i hver. Deretter pakker han 12 slike kartonger
i en pappeske. Hvor mange egg er det i pappesken? Vet ikke ☐

6 ☐ 18 ☐ 12 ☐ 72 ☐ 144 ☐ 60 ☐

Task 7: Calculate 7x26 =

Task 16: Lise returned seven 1.5-liter bottles and fifteen 0.5-liter bottles. For each 1.5-liter bottle, she gets NOK 2.50 and for every 0.5-liter bottle, she gets NOK 1. How many NOK does Lise get for all the bottles?

Task 19: A group of friends meets for a movie night. Each person eats half a pizza. In total, they ate six whole pizzas. How many people were involved in the movie night?

Task 28: The parents of Johan run a chicken farm. Johan is packing eggs in cartons that holds six eggs each. Then he packs 12 such cartons in a cardboard box. How many eggs are

Figure 1: The four different tasks. The number of the tasks indicates the tasks number in the test

Task 16 is a multistep WP with decimal numbers. The situation in the task is familiar for most students and is a money-problem, which is often seen to be easier than other contexts. There are almost 30 words in the text, some of them “unnecessary” decimal numbers, which makes the WP more complicated. Of students in the SRA group (both 5th and 6th grade), only 57 % answered correctly on this task. Making this the problem where students most frequently respond incorrectly. In LRA, 38.5 % and 44 % of the students in 5th and 6th grade, respectively, answered correctly on this item. There is a significant difference between the SRA and LRA students in 5th grade ($p<0.01$), but not for 6th grade ($p=0.03$). Like Daroczy et al. (2015), we cannot conclude that the reason for students answering this task incorrect is that the linguistics are more difficult or the task more arithmetical complex. This task might also be outside the curriculum for 5th and 6th graders. Whatever the reason, students with SRA scored better than students with LRA.

	5th grade				6th grade			
	SRA		LRA		SRA		LRA	
Task number	N	%	N	%	N	%	N	%
7 (Calculation)	343	61,5	121	52,5	464	77,8	72	65,3
16 (Bottles)	341	57,2	122	38,5	465	57,4	75	44,0
19 (Pizza)	339	77,0	119	63,9	460	83,0	75	66,7
28 (Eggs)	341	86,2	120	65,0	462	91,8	71	64,8

Table 1: Percentage (%) of students in our population (N) that answered the different task, split by SRA and LRA

Both task 19 and 28 are single step arithmetic WPs with no extra numbers in the text. This makes these two WPs easier for the students to solve correctly. Task 19 comes from a familiar situation for students, but it has a twist. It is more common to know how many people there are, and then find out how much you need to buy. Here it is the other way around. The questioning in itself can contribute to make this task more difficult. To solve it, the students have to work with decimals. Task 28 is probably derived from an unknown situation for most of our students. Although this item is from an unknown context, it is still the easiest because it only consists of multiplication with known numbers and integers. On both of these two, there was a significant difference between students in the SRA and LRA group ($p<0.01$) both for 5th and 6th grade students.

Furthermore, there are more students with SRA answering correct on the last two of the WPs (task 19 and 28) than on the control task (7). This indicates that reading text does not need to be an obstacle, it can also be a help for the students to get the right answer. It looks like the text and the context actually can help the SRA students. We find the same pattern for the LRA students in 5th grade, but not as strong. For the students in 6th grade with LRA, approximately 65 % responded correctly on these three tasks.

On all three WPs, there are a significant difference between students with LRA and SRA. This result indicates that reading abilities affects student’s ability to do mathematics, just as Nordtvedt (2010) stated. In our study, students with SRA are doing better on all task, except 16, (on 7: $p<0.01$, 19: $p=0.03$ and 28: $p<0.01$).

So far, we have looked at the result for 5th and 6th grade separately. Is there progress from 5th to 6th grade for students with SRA and students with LRA? If we take a closer look at task 7, we find that 61.5 % of the students in the SRA group in 5th grade and 77.8 % in 6th grade answered it correctly, which is a significant better result ($p<0.01$). Also on the task 19 and 28 there are a significant difference between 5th and 6th grade for this group of students ($p=0.03$ and $p=0.008$), but not for task 16 ($p=0.95$). For students in the LRA group, there are no significant difference between 5th and 6th grade on any of the task (p between 0.09 and 0.98).

As noticed, the 5th graders perform better in two of three WPs than in the calculation task. This is another factor that indicates that it is not necessary the text in the WPs that are the difficulty.

Actually, it looks like students with LRA can have a good informal mathematical understanding and have difficulties with doing the calculation/algorithms. By taking a correct answer on task 7, as an indication that students can multiply, is there then a difference between students with SRA and LRA when it comes to solving WPs? By picking out only those students who have a correct answer to task 7 (Table 2), there are a significant difference between the SRA and LRA students in 6th grade on task 19 and 28. On task 16 the difference is not significant (p=0.3). For students in 5th grade there are no significant difference between the SRA and LRA groups (p between 0.14 and 0.49).

As indicated before, it might be that the text actually helps the students with LRA, just like it helps the students with SRA. Another interesting finding, is that we cannot find the same difference in sixth grade. The student in 6th grade with SRA have an improvement form task 7 to task 19 and 28, but not the students with LRA. According to the curriculum, the algorithm for multiplication is introduced during 5th or 6th grade. This might explain these results. If that is the case, it makes the findings in 5th grade even more significant since the context helps students with LRA to do more advanced calculation than they can do without context. The same applies for the 5th graders with SRA.

Our result show a difference in percentage of students with SRA and LRA in solving task 7 correct although this is not a WP (significant difference for student in 6th grade, but not for students in 5th). This point toward that students with SRA also do it better than students with LRA on task there reading is not a primarily part.

	5th grade				6th grade			
	SRA		LRA		SRA		LRA	
	N	%	N	%	N	%	N	%
16 (Bottles)	192	68	56	57	330	66	43	58
19 (Pizza)	207	80	63	76	352	87	46	70
28 (Eggs)	206	90	61	85	351	97	44	86

Table 2: List of how many of our population that answered task 7 (calculation) correctly (N), and how many percentages of these that answered the corresponding task correctly (%). Divided in SRA and LRA

On one-step WPs, there are a significant difference between students with SRA compared with those with LRA. The reason for this might be that students with LRA have problem making a visual-schematic representations (Boonen et al., 2013). Both task 19 and 28 should be easy to make such a representation. Our result indicate that it does not seem that unknown context is as important as calculation with decimals or integers for students with SRA since task 28 are easier than task 19. For students with LRA are roughly equal numbers of students who answer correct to both of these two tasks. By looking only at students that are solving task 7 correctly (Table 2), we find that there are more students solving the task with whole numbers (28) correctly, than the task with decimal numbers (19).

Closing remarks

In our study, a greater proportion of SRA than LRA students answered correctly on all three WPs. This is also the case for a task with no text.

Another interesting result is that among students in 5^{th} grade there are more students answering correct on two WPs than there are students answering correct on the calculation task. This might imply that the text, also for those students with LRA, can be a help in solving multiplication problems when students are not completely confident in multiplication. When students are more confident in multiplication, this assistance in the text are not as prominent.

Nordtvedt (2010) concludes that there is a strong positive correlation between numeracy and literacy, our data supports Nordtvedt findings. Like Daroczy et al. (2015), we conclude that LRA students' difficulties with WPs are both linguistic and numerical. Our data implicates that the LRA students have bigger difficulties with mathematics all over, and not only WPs.

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