Stockholms universitet

What do students' feel about mathematics?

Compulsory school students' emotions and motivation towards mathematics

Martin Nyman

Licentiatuppsats
Rapporter i matematikämnets och naturvetenskapsämnenas didaktik
Nummer 16, 2020

Rapporter i matematikämnets och naturvetenskapsämnenas didaktik
Nummer 16, 2020

What do students' feel about mathematics?

Compulsory school students' emotions and motivation towards mathematics

Martin Nyman

Stockholms universitet

What do students' feel about mathematics?

Compulsory school students' emotions and motivation towards mathematics

Martin Nyman

ISBN 978-91-7797-988-3
Tryck: Arkitektkopia AB, Stockholm 2020
Distributör: Institutionen för matematikämnets och naturvetenskapsämnenas didaktik

Summary

This licentiate thesis deals with compulsory school students' expressed emotions and motivation towards mathematics. Theoretically, it has been guided by Hannula's meta-theory on affect (e.g., 2012), of which emotion and motivation are part. In this thesis, emotion is defined using models from Schirmer (2015), and motivation correspondingly by Shunk et al. (2010). In the analysis, a model for emotion developed by Lövheim (2012), and models for motivation proposed by Ryan & Deci (e.g., 2000) and later further developed by Sumpter (2012), were adopted.

This thesis focuses on two studies. In the first study, in search of nuanced knowledge about students' experiences of mathematics, a primarily qualitative approach was adopted in interviews conducted with 19 primary school students. The results reported in Paper I (Nyman & Sumpter, 2019) confirm previous research which found that students express both intrinsic and extrinsic motivation for doing mathematics. But the results also indicate further motivational nuances, and I propose a division of each dimension into six subcategories. The results reported in Paper II (Nyman, in press) indicate that students' negative emotions towards mathematics are directed towards themselves, as shame or distress, and not externally as anger. The results also indicate that there are connections between emotion and other affective concepts such as motivation and social dimensions, but also more technical aspects, for example, being allowed to listen to music.

The second study, reported in Paper III (in preparation), gathered questionnaire data from 222 grade 8 and grade 9 students, and aimed to compare differences between the two grades as well as between boys and girls. The results show that the only significant difference between boys and girls is on issues relating to motivation: Girls generally express being more extrinsically motivated to doing mathematics than boys.

List of included papers

I Nyman, M., & Sumpter, L. (2019). The issue of ‘proudliness’: Primary students’ motivation towards mathematics. *LUMAT: International Journal on Math, Science and Technology Education, 7*(2), 80–96–80–96. https://doi.org/10.31129/LUMAT.7.2.331

II Nyman, M. (in press). Primary students’ expressed emotions towards mathematics. In *Sustainable mathematics education in a digitalized world. Proceedings of MADIF 12*. SMDF.

III Nyman, M. (in preparation). Secondary students’ expressed emotion and motivation towards mathematics.

Acknowledgments

I started this project four and a half years ago. During this time, the help and support of numerous people have been absolutely crucial in enabling me to complete this work. I want to begin this thesis by thanking these wise, generous and skilful people.

First, Stockholms Stad for providing the financial means for this project. Working as a teacher while pursuing postgraduate studies in subject didactics is a model that is mutually beneficial for both activities; at least, that has been my experience. I would also like to express gratitude to my principals, teacher colleagues and co-workers (a lot of people!) for all manner of support they have given me during this time.

Without the guidance and support of my supervisors I would never have reached any destinations on this journey, and certainly could not have completed this thesis. Paul Andrews, Kerstin Larsson and Lovisa Sumpter, I am forever grateful to you for generously having shared your insights with me. Especially Lovisa, who stood by me and endured throughout this time, and even succeeded in dismantling my stubbornness. I also want to thank Lisa Björklund Boistrup, Hendrik van Steenbrugge and Laura Tuohilampi for the guiding lights they lit by reading and commenting on earlier drafts of this thesis. And to crown it all, a big thank you to Peter Liljedahl, my fantastic opponent.

My steady companions in the MaRCU seminars – under Kerstin Pettersson and Lovisa Sumpter's guidance – were Johannes Berggren, Abraham Beyene, Helena Eriksson and Karin Landtblom, who contributed to fruitful discussions on a variety of research-related topics. I am grateful for their input on my work as well as for the opportunity to learn from them when they shared their own work. The value of this type of reciprocal sharing environment can hardly be overstated, and, for me, this has been the place where I slowly began to regard myself as a researcher.

At this point, I want to mention people I meet and work with almost every day, and whom I have come to call my friends. Since it would feel odd to speak to them in English, I now shift to Swedish. Låt mig börja med att nämna STLS. Därför att det har varit platsen där identiteten som forskande *lärare* vuxit fram. STLS har också varit platsen där relevansen av den här typen av undervisningsnära forskning känts starkast. Först vill rikta ett allmänt tack till hela STLS och dess vetenskapliga ledning, som under min huvudsakliga tid utgjorts av Maria Andrée och Inger Eriksson. Sedan vill jag tacka mina kamrater i matematiknätverket för ni är just det; kamrater, och därmed ovärderliga! Attila Szabo och Laura Caligari Moreira, det är ett nöje att få arbeta tillsammans. Och mina

tidigare kamrater i nätverket; Johannes Berggren, Verner Gerholm, Jenny Fred, Anna-Karin Nordin och Sanna Wettergren, jag saknar er.

Eftersom jag har gått en forskarskola så har jag ju haft ”klasskamrater”; Mattias Björklund, Sebastian Björnhammer, Björn Kindenberg, Anette Resare Jansson, Laila Riesten, and Anna Wallin. Vi anträdde den här färden samtidigt och med vingliga steg, men tack vare er kände jag hela tiden en trygghet i gruppen. Jag vill rikta ett särskilt tack till Mattias och Björn; tryggare vänner finns nog inte.

Allra sist kommer det tveklöst allra viktigaste tacket, det till min familj, Ewa, Isak och Julia. Kort sagt, ni betyder allt för mig, och utan er hade *inget* av det här avhandlingsarbetet varit möjligt. Ni har bidragit med förbehållslöst peppande, skarpögda läsningar av mina texter och initierade samtal om teori. Så kram och tack för allt det och förlåt, för alla stunder framför datorn, för alla stress-snäsiga svar, för alla hämtpizza-middagar, och för alla gånger mina tankar vandrat iväg till något transkript eller någon forskningsartikel mitt i ett samtal.

Nu betraktar jag det här arbetet som klart, ja nästan i alla fall, och jag ser fram emot att arbeta som just *forskande* lärare.

Norra Djurgårdsstaden, Stockholm, november 2020

Martin Nyman

Contents

Introduction

"Affect [is] at the centre of learning, alongside cognition" (Lewis, 2013, p. 74).

Every teacher can testify to the fact that there is a lot going on in a classroom. With a didactical research interest, you can find a vast number of potential research angles from almost any lesson or school activity. This thesis is interested in the affective dimensions of students' emotions and motivations towards doing mathematics, how these issues connect to each other, where they come from, and how they differ and evolve. We must assume (like Lewis) that these issues matter for student learning, both in the short and long term. From a subjective perspective, I want to start with a short personal anecdote from one of those eventful classrooms.

I was teaching a class of seventh graders arithmetic and I wanted the students to take a test. After having done the test they were instructed to switch with a friend and correct each other's work, aided by an answer sheet. Suddenly, I was eagerly called upon by two students who had started to check each other's answers, and who claimed that there was an error in one of answers. The task was: "Find a number between 2.9 and 2.10". Since there is an infinite number of correct answers, the answer section acknowledged this by providing two examples: 2.7 or 2.12. Neither student could accept either of these two examples as correct. After being given the explanation that 2.10 is actually the same value as 2.1, one of the students sighed, "(a)h… yes… of course!" while the other did not even indicate whether or not she had understood the explanation; instead she was very frustrated, complained that the task was "unfair", and that there should have been a "warning" at the top saying: "beware, this test may contain problem-solving tasks!"

From the practice perspective of teaching, the interesting issue illustrated by this story is not the type of task the students had done, nor the mathematical understanding they displayed, or what type of teaching they had been offered up to this point. The issues I want to focus concern the emotions and motivation towards mathematics and mathematics education that this story illustrates, and the overall affective impact on the situation. More precisely, which emotions and feelings, and what type of motivation do students express in relation to mathematics? What triggers them? And how are they connected?

Accepting that affective dimensions are an integral part of thinking (Goldin et al., 2011), the importance of affective factors in learning should not be underestimated (Evans, 2000; Lewis, 2013; McLeod, 1992). The link between affective factors and student achievement is stronger than any other external factor, such as socio-economic background or cognitive skill (Suárez-Álvarez

et al., 2014). In terms of long-term effects, the link between motivation and achievement and its impact on education and career choices differs between genders (Stoet & Geary, 2018).

In addition, interaction occurs at several different levels: subconstructs within the affective domain such as emotion, motivation and beliefs interact with each other within the individual; but also, based on physiological, psychological and social factors, they interact between individuals in culturally and socially developed environments. The compulsory school is an example of such an environment where the child becomes a person (Lave & Wenger, 1991). Finally, there is an interaction with the mathematical content as well. An individual's affective disposition towards the content shapes their way of accessing and processing it (Hernandez-Martinez & Pampaka, 2017).

Despite large amounts of research on affect in mathematics learning, Hannula laments the current state of affairs in the 2019 ESM (Educational Studies in Mathematics) special issue: "How and when does mathematics become a source for anxiety or pride?" (Hannula, 2019, p. 309). Seeing that affect is an important dimension of mathematics thinking, and furthermore acknowledging that it is also a product of learning that in turn predicts future behaviour and choices stresses the need for finding answers to Hannula's question.

Thus, the overarching purpose of this thesis is to contribute with nuanced knowledge about students' emotions and motivation in relation to mathematics and mathematics education, and to discuss emotion- and motivation-related differences between students of different ages over the considerable time span of compulsory schooling. Articles I and II focus on primary school students; the analysis is mainly qualitative and addresses the following research questions:

1.1 What different types of motivation do primary school students express? and

1.2 How are these different types of motivation interrelated in the students' responses?

2.1 How do students express emotion in relation to mathematics and what are the characteristics within these expressed emotions?

2.2 How, and by what mechanisms are students' emotions linked to expressed motives? Article III three focuses on lower secondary students and the analysis is mainly quantitative, addressing the following research questions:

3.1 How do students express their emotions towards doing mathematics?

3.2 How do students express their motivation for doing mathematics?

3.3 How do motivation and emotion interrelate?

3.4 What gender differences, in relation to questions 3.1 and 3.2, can be found?

These research questions are discussed more thoroughly in the articles; the discussion in this thesis therefore attempts to condense the overall results and suggest some implications for practice.

Background

The studies in this thesis have been guided by Hannula's meta-theory on affect (2011, 2012, 2019). Hannula establishes that affect can be considered an umbrella concept consisting of sub-constructs (attitude, anxiety, beliefs, meaning, self-concept, emotion, interest, motivation, needs, goals, identity, norms, values) with a varying degree of stability where emotion is one of the more stable. However, even though emotion seems to have some sort of interconnection with other constructs, Hannula shows that the various other interconnected concepts are differently defined in different theories and also that they differ in scope – whether, for example, social interaction issues are addressed or not. Hannula's meta-analysis resulted in a model that pays attention to three dimensions with decisive influence on affect:

1. cognitive, motivational and emotional aspects of the affect;
2. rapidly changing affective states versus relatively stable affective traits; and
3. the physiological (or embodied), psychological and social nature of affect.

(Hannula, 2012, p. 143)

The three dimensions are presented in a cube-like matrix. Hannula emphasises that the model should be regarded as an illustration of which constructs are in play when considering affective issues, not as an instrument for defining the primacy of one construct over another or measuring the ratio between them.

Motivation

Motivation can be understood as our driving force for action, the engine that keeps us going. In Ryan and Deci's (2000) words it is about the person being energised towards an end, thus connecting the motivation to some goal. This is made explicit in the definition by Schunk et al. (2010): "Motivation is the process whereby goal-directed activity is instigated and sustained" (p. 4). This is the way motivation is defined in this thesis. Important to remember, however, is that motivation can be either positive or negative; a person's goal can be avoidance as well as attainment (Sekreter & Serin, 2017). This valence is one motivational dimension, connected to the "direction" in the quote above; however, we also need to consider its origin, from whence it is instigated. This is often less easily spotted and due consideration must be given to whether the person is intrinsically motivated (driven by inner forces like a need or want), or rather extrinsically obligated and coerced into action by factors in their environment. The division into these two fundamentally different origins of

motivation points to the general need to discuss differences in motivation not only in quantitative but also in qualitative terms.

Another aspect of motivation concerns its dimensions. One way of looking at motivation is as a linear continuum. It emanates from the inherent control within the individual that creates intrinsic motivation, and ranges, through a number of externalising steps that move the control ever further out from the individual, creating increasing levels of extrinsic motivation, to finally conclude in amotivation – a state where the individual experiences a lack of control, resulting in non-action (Deci & Ryan, 2000; Ryan & Deci, 2000). This study, however, takes a more two-dimensional approach, describing intrinsic and extrinsic motivation as two different constructs in different relations with each other that the individual can experience (Amabile et al., 1994). This view accepts that a person can experience a mix of both types of motivation at any given time; the detail is described in sub-scales: intrinsic motivation is divided into Cognitive and Emotional aspects, with extrinsic motivation correspondingly divided into an Outward and a Compensation aspect (Sumpter, 2012).

Several studies investigate connections between achievement and motivation. For example, Tzohar-Rozen & Kramarski (2014) discuss the emotion-motivational dimension in self-evaluation aspects in problem solving. Further, a study by Desoete et al. (2019) looking at the relationship between motivation and achievement reports that intrinsic motivation is connected to accuracy but not to fluency. Their results also show that poor performers are less intrinsically motivated and more prone to overestimating their achievements. Samuelsson (2011) shows that even though intrinsic and extrinsic motivation[1] are strong achievement predictors, the individual's self-concept is even stronger. This trend is even more marked in Sweden and the Nordic countries than in the rest of the OECD countries.

Another line of inquiry concerns how beliefs are connected to motivation and how this can affect learning (e.g., Rojo Robas et al., 2018), and, by extension, academic and career choices (e.g., Rodríguez et al., 2020). Ramirez (2017) suggests that students can be motivated to forget as a way of reducing threats, and not only by reducing mental focus on irrelevant issues, but also on factors connected to deeper personal views about the self.

In educational situations it becomes necessary to clarify motivational ownership and direction – who is motivated, is it the student or the teacher? And for what? As Lave & Wenger (1991) point out:

> When the process of increasing participation is not the primary motivation for learning, it is often because "didactic caretakers" assume responsibility for motivating newcomers. In such circumstances, the focus of attention shifts from co-participating in practice to acting upon the person-to-be-changed. (p.112)

[1] Samuelsson uses the terms internal and instrumental motivation, respectively.

Thus, it would be more valuable for teachers to focus on designing a type of teaching that enables students to participate in learning situations than trying to motivate them to learn. Instead, teachers are often trapped in the conflict between "learning to know and learning to display knowledge for evaluation" (Lave & Wenger, 1991, p. 112), the effect being that the focus of educational activity changes from learning to assessment (Hultén, 2019). This shift has not developed in isolation; in western societies a general culture of measurement, that has perverted into focusing what is measurable rather than what is desirable, has gained strength during the last decades (Biesta, 2011; Bornemark, 2018). In a review of research carried out in Sweden, Giota (2013) points to problems in part connected to this personal motivation versus systems goals conflict:

> However, enthusiasm and intrinsic motivation seem to get the least attention at school. /…/ Instead, joyful learning seems to be replaced with learning where students are expected to be more future oriented and to use learning as a means to achieve future goals, which in turn can demand certain sacrifices here and now that not all students adhere to or manage to fulfil. (Giota, 2013, p. 155. [My translation])

Some research has focused on this relationship between motivation and assessment, and the results often show emotional connections, primarily to anxiety (Ramirez et al., 2018), and often associated with tests (Lewis, 2013). There are also indications that students' motivation for mathematics decreases during the compulsory school years (e.g., Blomqvist et al., 2012; Vinni-Laakso et al., 2019).

A person's inherent drive for self-determination in situations they participate in can be problematic considering the overall goals of public education; qualification, socialisation and subjectification of the young generation (Young, 2015). Further, Young (2015) makes a helpful division of these goals into two categories: context-dependent and context-independent learning. Context-dependent learning is spontaneous learning, primarily aimed at social functionality; for example, a child learning his or her native language. Context-independent learning is fundamentally different:

> In contrast, context-independent learning, which had a highly restricted domain in most societies, is difficult and far from spontaneous; it takes place at uneven rates, depending on the circumstances of the learner as well as on what is learned; it can be experienced as alien to experience and be resisted; it takes a diversity of forms and, most fundamentally, it requires the involvement of specialist teachers and the creation of specialist institutions. The difficult task of these specialists is, through their pedagogy, to create contexts (usually in schools, colleges and universities) which free learners from their everyday contexts and enable them to think conceptually. (Young, 2015, p. 18)

Described this way, context-independent learning must be considered one of the fundamental tasks of public education. Further, and for teaching practice purposes perhaps more importantly, it illustrates an inherent, and inevitable, root of potential conflict that many educators recognise from struggling with trying to motivate reluctant students.

Emotion

Emotion has been studied from the perspective of its physiological effects, i.e., the measurement of eye movements, heart rate, skin conductivity, hormonal levels, etc. For educational purposes, these values often need to be translated into descriptions of a person's feelings or wellbeing. As a complement to this type of research, emotion has also been studied through more qualitative and holistic methods, often using questionnaires and interviews in different forms. These methods are based on self-reports and raise issues of differences between having an emotional sensation, feeling an emotion, thinking about this feeling, and describing that feeling. Lewis (2013) is frustrated by this lack of definition:

> Despite this early attention, emotion seems to hold an unusual place in the literature on mathematics education, and even in the literature on affect in mathematics education. Whilst it gains attention as a topic of interest, there is little systematic or detailed data on emotions, and alongside this, the construct is not often fully theorised. (p. 72)

From the publication of the first theory of evolution, emotion has been described as an evolutionary product with great importance for the individual's survival in general but more importantly, for assuring us of our special place among living organisms with our inherent interconnections between the cognitive, affective and social functions (Damasio, 2004, 2018; Maturana & Varela, 1992). From a psychological standpoint, descriptions of the role and function of emotion differ. To date, the vast field of empirical research and philosophical investigations of the concept of emotion can be divided into three main approaches (Schirmer, 2015): one is the categorical approach that sees emotion as a finite number of discrete entities that evolve in the individual over time. Two is the dimension approach that describes emotion as the output effect of the individual being moved from their core state in a two-dimensional space defined by pleasure–displeasure and activation–deactivation. Third, the appraisal approach delineates emotion as a sum, or the outcome of a number of appraisals of (mainly) external situations or stimuli – the individual constantly and autonomously appraises the situation around them, and the weight of these appraisals triggers an emotion.

Definitions in this thesis follow the appraisal approach and build on the way the biochemical mechanism of human emotion onset is described by Damasio (2000). This mechanism is divisible into five steps: the first is the initiation of the emotion, made by some sensory input. It is important to stress that this input can

have an internal or an external origin. The following two steps include response patterns in the form of the transmission of substances that produce transformations in the body and brain. The two last steps describe the transformation process when the emotion is made conscious, or when the emotion becomes a feeling. For educational purposes, this complex mechanism highlights two matters of importance. One, that emotions can be strong even when unconscious. Two, that the possibility of an internal origin (e.g., a student's body being stressed by a physiological deficit) enables potential interconnections with external factors (e.g., being stressed for time in an exam) leading to mismatches between input and output. This thesis defines emotion as "conscious or unconscious mental states elicited by events that we appraise as relevant for our needs and that motivate behaviours to fulfil these needs" (Schirmer, 2015, p. 26). From neurological models we understand that emotional sensations (generally affective) may not necessarily be experienced consciously, and may, therefore, not necessarily be expressed.

When discussing emotions towards mathematics it is difficult to avoid talking about anxiety (Rodríguez et al., 2020), Lewis (2013) even means that anxiety is "the most frequently mentioned aspect of emotion" (p.72). Worry about mathematics can be defined as "feelings of tension, anxiety, or stress that occur in situations that involve math problems" (Trezise & Reeve, 2017, p. 134). There seems to be a link between worry, mathematics anxiety and working memory capacity that is bidirectionally detrimental. A consequence of this is that students experiencing high levels of mathematics anxiety tend to develop a pattern of avoidance. This avoidance can manifest in students sacrificing accuracy for speed in performance (Trezise & Reeve, 2017), or actively forgetting content matter previously learnt (Ramirez, 2017). With all its negative effects we must sadly admit that feelings of mathematics anxiety are common and increase during early adolescence (Trezise & Reeve, 2017). Furthermore, boys and girls experience mathematics anxiety differently, with several studies suggesting that girls generally express higher levels of mathematics anxiety, even when achieving on par with or better than boys (Pekrun et al., 2017; Ramirez et al., 2018; Rodríguez et al., 2020). Both genders show an increase in mathematics anxiety levels over the compulsory schooling period, with the steepest increase during the primary school years (e.g., Madjar et al., 2018).

As a concluding remark, a host of research acknowledges the interconnections and interplay between motivation and emotion: the connection between emotion and motivation (Sutter-Brandenberger et al., 2018); the reciprocal effects of emotion and achievement (Pekrun et al., 2017); the connection between motivation and achievement (e.g., Desoete et al., 2019; Liu & Hou, 2018; Rojo Robas et al., 2018); and explicit cognitive dimensions in emotions and motivation in problem-solving (Baten & Desoete, 2019). However, a discussion about how teaching strategies can respond to the dynamics of motivation and emotion in learning situations is rare.

Social, Physiological or Psychological interaction

A human being can be described as an autopoietic system (Maturana & Varela, 1992) – a complex, dynamic system that interacts with the surrounding environment, including both inorganic and organic matter, in order to "survive". A key feature in autopoietic systems is their resilience; the system is dynamic and can adapt to changes by changing its own structure enough to persist (Hannula, 2012). Of relevance to this thesis is the implication here that the individual's affective dimensions – those that are inherently psychological – are constantly connected to social and physiological dimensions to form a person's actions and behaviour, hence also affecting learning and achievement (Järvenoja et al., 2017; Rodríguez et al., 2020). Eventually, students' affective experiences of mathematics will affect their career choices (Mujtaba & Reiss, 2016).

Adopting a didactical view entails narrowing the scope to focus on learning situations and to look at the function of social factors in learning. There appears to be a division between two paradigms – aquisitionist and participationist (Larsen, 2014; Liljedahl, 2015; Sfard, 2005). The former is sometimes referred to as "traditional" (e.g., by Sfard, 2005), and focuses primarily on the individual, describing learning as the process where the individual receives, processes, transforms and accumulates knowledge. The mechanisms are typically psychological in nature, and primarily cognitive. On the other side of the divide, the participationist view acknowledges the individual but focuses on the social environment in which the individual is situated and where learning occurs and emerges as a product of the joint labour of the group to which they belong. Here, the individual's will and possibility to participate are described in terms of motives that are socially, and thereby also both culturally and historically rooted (Arievitch, 2017; Davydov, 2008; Leontyev, n.d.). Hence, both paradigms describe an interconnectedness between motivational and emotional dimensions and other bodily functions as well as with social settings. However, the functions of motivation and emotion are fundamentally different when viewed through the different lenses of the two paradigms, respectively. And because they inform education differently, educators need to consider which of these views on learning they adhere to.

State or Trait–a temporal dimension

Emotional and motivational phenomena are sometimes discussed in terms of stability. McLeod (1992) discusses different affective phenomena as being distributed over a stability spectrum with an increasing level of stability from emotion to attitude to belief, but also differentiated in other ways: "… we can think of beliefs, attitudes and emotions as representing increasing levels of affective involvement, decreasing levels of cognitive involvement, increasing levels of intensity of response, and decreasing levels of response stability" (McLeod, 1992, p. 579).

Other researchers have since tried to differentiate between temporal dimensions. Hannula (2011, 2012) claims that affective phenomena must be studied in different timeframes; for example, a short-lived emotional state is different from a person's ever-present and slowly evolving emotional traits. Both these dimensions are parts of a person's self-concept (Samuelsson, 2011). Hannula (2011) laments that these temporal issues have seldom explicitly been the focus of research and that they are perhaps neglected, featuring only implicitly in the research on affect. Further, Lewis (2013) claims that emotion is often studied as a trait, and less often as a state, resulting in these short-lived emotional states being classified as something other than emotions.

To summarise the theoretical investigations so far, this thesis rests on a foundation of two concepts – motivation and emotion. These two concepts consist of a mix of three factors – psychological, physiological and social – that play out differently in different timeframes and with effects also varying over time. This mechanism leads to a complexity, resulting in individuals relating to these concepts with a varying degree of both clarity and resilience depending on their subjective experience of the concept's strength.

Gender

When investigating nuances in student expressions, issues of gendered biases as well as gender stereotypes become important since they can be expected to reciprocally influence and be influenced by personal experiences.

The term gender can refer to distinctions between different male and female attributes associated with biological sex; or to differences in the distribution of, for example, physical strength, attitude or influence between males and females as groups; or to discourses which are culturally rooted and intertwined with other discourses such as race, ethnicity or social class (Nielsen, 2017).

In education, the term "doing gender" (e.g., Bjerrum Nielsen, 2003; Wedege, 2011) becomes important. It deals with gender stereotypes being formed in social situations where people "actively 'do' gender in ways that are meaningful to them culturally as well as personally" (Nielsen, 2017, p. 27). According to Nielsen, gender is in part produced in school and is thus an important issue for educational research. Further, research points to substantial differences between the sexes regarding attitudes and emotions, as well as choices about higher education and careers (Alawi & Mubarak, 2019; McGeown & Warhurst, 2020). In short, it seems that there are differences between boys' and girls' relations to mathematics in almost every aspect apart from cognitive ability.

From national studies conducted in Sweden we know that gender differences indeed exist; while these differences are often found to be close to zero in relation to certain aspects, they are considerably significant in others. For example, Sumpter (2012) describes the most prominent differences being not between

girls and boys but between girls' descriptions of girls and girls' descriptions of themselves. A study by Frid et al. (2020) shows that a vast majority (76%) of girls do not assign mathematical knowledge to gender at all, compared to only 35% among the boys. But at the same time, both girls and boys believe that other people (peers, parents and teachers of both genders) to a lesser degree than themselves believe this, and they further believe that other people are relatively more prone to assign gendered differences to mathematical knowledge than themselves do. This points to culturally determined effects on boys and girls through gender stereotyping that potentially cause undesirable differences in the choices and possibilities available to girls and boys. A similar conclusion is reached by Eriksson et al. (2020) when they claim that, "(c)ultural values are pervasive and could influence almost every aspect of the academic environment of boys and girls" (p. 13).

Methods

The aims of this study are diverse. One aim is to contribute with detail and nuance about students' emotions and motivation in relation to mathematics and mathematics education. A second aim is to quantitatively describe differences regarding these issues between age and gender. Therefore, a combination of qualitative and quantitative methods was used. This dual aim is represented as two studies, 1 and 2, where Study 1 is primarily qualitative and Study 2 primarily quantitative.

Outline of the two studies

Table 1 summarises the general design of the two studies, including similarities, differences and timeframes.

Table 1

Distribution of studies

Study	Method	School grades	When	$n_{schools}$	$n_{respondents}$	Article nr
Pilot	Interview	2, 4, 6	Autumn 2016	3	15 (6 + 3 + 6)	–
1	Interview	2, 5	Oct-Nov 2017	3	19 (10 + 9)	I & II
2A	Question-naire	8, 9	Nov-Dec 2018	6	222 (78 + 144)	III
2B	Interview	8, 9	Jan-Feb 2019	6	24 (14 + 10)	–

In the pilot, the interviews were aimed at learning more about how to interview children. The interviews were semi-structured and allowed for improvisation on questions and follow-up questions. All these interviews were transcribed and used as a basis for developing the follow-up questions that would be needed in the interviews to come.

In studies 1 and 2B, a different interview guide was used. This guide was based on the questionnaire developed and used by Dahlgren Johansson & Sumpter (2010) and Blomqvist et al. (2012). The interviews in study 1 and study 2B were semi-structured. Study 2A was a questionnaire study, using the same questionnaire instrument.

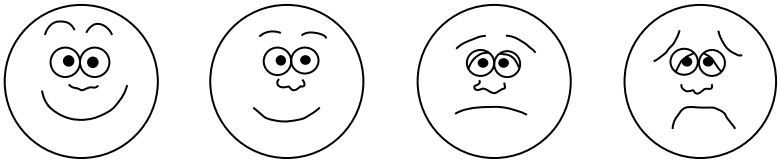
This variation of methods allowed me to make both quantitative and qualitative analyses.

Instrument

A seven-item questionnaire was used in two different ways: as a questionnaire for grades 8 and 9 and as an interview guide for semi-structured interviews for grades 2, 5, 8 and 9 (ages 8, 11, 14 and 15, respectively). This instrument was developed and used in two studies by Dahlgren Johansson & Sumpter (2010) and Blomqvist et al. (2012). Both studies looked at students in grades 2 and 5, and the analyses were mainly quantitative. The focus of my study 1 was students of the same ages and using the same instrument but with a mainly qualitative approach. The seven items, already phrased as questions, worked well as an interview guide. Further items 1–3 were designed for a scaled response using a four-step Likert scale with happy and sad faces (Figure 1).

Figure 1

Likert scale used in items 1–3



The items were:

1. What do you think about maths?
2. How do you feel before a maths lesson?
3. How do you feel before a Swedish lesson?
4. Why do you do maths?
5. How do you feel when you do maths?
6. What do you do when you do maths?
7. Please draw a picture of yourself when you do maths.

These faces worked even more efficiently as prompts than anticipated, and as interviewer I could then probe further for clarifications or examples (Kvale & Brinkmann, 2014). The following items were open-ended, complemented the initial questions and allowed respondents to go back to previous thoughts to both expand and adjust responses. In the last item, respondents were asked to draw a picture of themself doing mathematics.

After analysing the data from study 1, the research expanded and a second study was made, using the same questionnaire instrument with a quantitative focus but with later grades (8 and 9). This part is called study 2A. To enable an expansion of the qualitative data, a further choice was made to conduct the same type of interviews with the grade 8 and grade 9 students as had been done with the younger students in grades 2 and 5. This study is called 2B. However, due to the scope of this thesis, study 2B will not be considered.

Questionnaires

The seven-item questionnaire (see Appendix D) was distributed in hard copy to a whole class led by the mathematics teacher. This was typically during a mathematics lesson but occasionally in a science lesson or during a class meeting [Swedish: klassråd/mentorstid]. Data collection for each class followed the same pattern: I visited the class myself, and once their teacher had welcomed me, I informed the students that their participation was voluntary. Before administering the questionnaire, I briefly described my interest in students' motivational and emotional experiences with mathematics and then gently urged them to consider their responses carefully and privately. I did this hoping that it would render more sincere and comprehensive responses. Questionnaires were administered to five grade 8 and six grade 9 classes and it took around 30 minutes for each class to complete. The class teacher and I were present the whole time.

It must be noted that the Likert scale faces were originally chosen for a younger cohort than in this study, which, as anticipated, raised questions from some of the respondents – yet another reason for me to have been present during the process.

Interviews

Alongside questionnaires, interviews are probably the most frequently used method for gathering research data in the social sciences (Bryman & Nilsson, 2018). The structured or semi-structured interview, using open-ended questions, is a potent way of capturing nuances in people's attitudes, norms, opinions, and beliefs that is not possible with a questionnaire (Di Martino, 2019). Interviews

can generate different types of knowledge, for example, pragmatic information that describes how things work in certain situations (Kvale & Brinkmann 2014). Here, with the aim being to contribute with nuance, this pragmatic form of knowledge was the main focus.

I conducted the one-on-one interviews during what would otherwise have been lesson time. Teachers helped to find a suitable venue in the school building where the respondents felt relaxed and the interviews would not be disturbed.

After introductions and general information about the interview, including that it was voluntary and that participation could be terminated without notice at any time, I began by presenting the four Likert scale faces on four cards, respectively (see Figure 1). These cards were used as a starting point for the responses in items 1–3. For the last item, the drawing task, respondents were offered a set of coloured pencils and instructed to take as much time as they needed. This led to the interviews being quite different in length – between ten and 40 minutes but averaging around 20 minutes. Each interview was audio-recorded.

The students in grades 8 and 9 had, prior to the interview, answered the questionnaire, so they were not asked to do the drawing task in the interview. To avoid respondents merely repeating what they had already said in the questionnaire, roughly one month was allowed to pass before the interviews were conducted. For the same reason, these respondents were not presented with the face cards. The intention was not to test if the responses differed but to extract nuances and detail that the questionnaires had not revealed.

Sampling and participants

The sampling process was meant to be carried out in three steps: 1) selecting participating schools, 2) selecting participating teachers within these schools, and 3) selecting the respondents, i.e., the students. The selection of schools was limited to the municipality of Stockholm, mainly for the convenience of reducing travel. When the work began, it turned out to be much more difficult than anticipated to gain access to schools, simply because school principals felt that "their" teachers already had a heavy workload and declined to participate. This resulted in shifting the selection process to starting with step 2 because it was easier to contact teachers that I had a personal connection with and with their help gain access to schools. This can also be considered a type of convenience sampling.

Sampling of students in the pilot and in studies 1 and 2B was done in two steps: the first step involved collecting signed consent from parents or caretakers for all students below the age of 15 (n = 102). In the second step, the teachers were asked to pick out, among the volunteering students, individuals that were considered neither extremely proficient nor having grave difficulties

with mathematics. Study 2A only involved the first step. Note that this does not apply to the 144 grade 9 students since they had all reached the age of 15 and could therefore sign the consent form for themselves.

Ethical considerations

The ethics of research involves several considerations in relation to the respondents: providing information about the study without pre-empting the investigation or biasing the results, being careful with every participant's time without hurrying, encouraging people to participate without forcing or persuading them, ensuring that the respondent understands the questions without putting words in their mouth, probing the respondent for further/deeper explanations without intimidating them or persuading them change a position to suit the researcher's desired answer. I tackled these issues by constantly reminding myself of them, and by making sure that I was the one providing information, administering the questionnaires and doing the interviews.

In addition to the procedures described in this chapter, the ethical rules and praxes stipulated in Codex (Vetenskapsrådet, 2017) were followed. Thus, participants had signed consent (see Appendix A, B and C) from parents when necessary, and were informed both about the purpose of the study and that participation was voluntary and could be terminated without explanation by the participant at any time. All questionnaires were completed anonymously, pseudonyms were assigned during transcription and all data were handled and stored securely. Participants were also assured that no personal information would be shared with anyone outside the research team.

Limitations

Recognising the complexity in this field, this study should have benefitted from a methodological triangulation, for example, following the model in Dobie (2019) where student questionnaires were followed up with classroom observations as well as student interviews. However, due to the scope of this study a larger sample of interviews was considered a priority over classroom observations. The questionnaire study carried out in grade 8 and 9 were in fact followed up with interviewing a subset of the questionnaire respondents, but the results of these interviews are, due to time constraints, not a part of this thesis.

Interviews are always accompanied by several potential biases. On an epistemological level it is important to note that the studies upon which this thesis is built all follow the same discursive pattern where all data consist of respondents (the students) telling stories about themself to the researcher (Heyd-Metzuyanim,

2019). It was difficult to find methodological literature on interviewing children for educational purposes. Studies that examine dimensions of objectivity concerning interviews with children are mainly done for medical or diagnostical purposes to assess diagnose–treatment relations, or for forensic or other judicial purposes to ascertain levels of witness credibility. One important issue when conducting interviews is the asymmetric power relations between interviewer and interviewee (Bryman & Nilsson, 2018; Kvale & Brinkmann, 2014). This problem is potentially exacerbated when interviewing children in their school environment. Aiming to minimise this factor, I allowed some time for every interviewee to adapt to the situation and made sure that they felt comfortable participating. Throughout the interviews I strove to follow the interviewee's pace and line of thought as far as possible without losing track of the aim of the study.

The participants were convenience-selected for practical purposes; mainly to reduce travel. This limited the geographical area where the data were collected. However, guided by the socio-economic school index published by the Swedish National Agency for Education (Skolverket, 2018) it was possible to find a reasonable diversity of background factors for the quantitative study. With regard to sampling, it can be noted that it was challenging to get schools to participate and that I had to rely on personal contacts to gain access to schools.

Method of analysis

The analytical methods adopted in relation to the various facets of this study are more thoroughly described in the articles. No additional analysis is made in the thesis; instead, this chapter serves as a summary of the analyses made, following the main objectives of the three articles, respectively. This chapter also attempts to expand on a discussion of the methods chosen.

Qualitative analysis

The data from the interviews were thematically analysed (Braun & Clarke, 2006) following several iterative steps – initial steps adopting deductive principles and concluding steps adopting inductive principles, both described below.

I personally transcribed all the interviews prior to analysis. Transcriptions were made verbatim, following standard Swedish spelling, and utterances where the respondent faltered were corrected in the transcripts. The typical example in Swedish is around adverbs of time: "Yesterday, I was late for school" would be the correct form in English but in Swedish the correct form is (translated) "Yesterday was I late for school". Further, various sounds common in speech but without any real meaning, like "ehrmm…", were generally marked with <thinks> or <pauses>. Extended pauses, longer than about 2 seconds were also marked <pauses>.

Below, follows a brief description of the analyses made in relation to motivation and emotion published in articles I and II, respectively. A first and overarching analytical step was concentrating the analysis on passages in the interviews relating to specific questions. Therefore, the analysis focusing on motivation was done on the basis of question number four: "Why do you do maths?" and its follow up-questions. Correspondingly, the analysis focusing on emotion drew on questions number one (What do you think about maths?), two (How do you feel before a maths lesson?) and five (How do you feel when you do maths?) and the conversations following them. As a second step, each interview was examined for instances where the interviewee returned to a topic previously touched upon, for example, motivation, in a previous part of the interview. This way the questions in themselves constituted a first deductive step of the analysis.

Motivation – Paper I

In accordance with the analysis for emotion, the analysis for motivation started with marking the transcripts where respondents made reference to goals, intentions, activity, engagement, or any similar expression of motivation (Schunk et al., 2010; Sekreter & Serin, 2017). Then the responses were connected to one of the two main themes, extrinsic or intrinsic motivation (e.g., Ryan & Deci, 2000). Responses connecting to reward or punishment from the outside world were coded as extrinsic motivation, like "[if I can't do it] it will be embarrassing". An instance when a respondent expressed interest or satisfaction was coded intrinsic motivation.

In the second analytical iteration, data from the extrinsic motivation category were mapped against two subthemes: Outward, here understood as social-gain values, and Compensation which is concerned with personal-gain values (Amabile et al., 1994). Accordingly, data connected to the Intrinsic motivation category were mapped against the subthemes Cognitive and Emotional (Sumpter, 2012), framing knowledge and personal development, or emotional arguments, respectively. The third and last step of the analysis followed the procedure of inductive thematic analysis described under emotion.

Generally, in the cases where a respondent's statement contained more than one theme, the utterance was split.

Emotion – Paper II

First, the transcripts were marked where the responses contained expressions of emotion, both in relation to items explicitly framing emotion, (e.g., item two: "How do you feel before a maths lesson?") and throughout the interviews. The marked instances were then coded positive, negative, or neutral. For example, "I like math when …" was coded positive, and "(i)t makes me stressed when …" was coded negative, and instances where the respondent referred to factors that were emotionally neutral, such as hunger, were coded neutral. In the second step, the utterances were re-examined through the lens of the basic emotions described by Lövheim (2012). For example, "I feel relaxed" was coded under the subtheme Relief, and "fun", "like", "happy" were coded under the subtheme Joy. This example also illustrates a decisive difference in emotional strength or intensity where "happy" is considered to be stronger than "like".

Further, the framework developed by Hannula (2012) describing different factors related to affect was adopted to expand the analysis beyond the descriptive. The coding focused on how the respondents expressed the origin of or reason for a specific emotion. For example, "I'm challenged by it" connects to the theme Motivation, and "I don't want my friends to laugh" connects to the theme Social. Inherent in each of these themes is a (potential) range from positive to negative. Lastly, an inductive search, following the approach of thematic

analysis (e.g., Braun & Clarke, 2006), was made. This was a recursive search for criteria creating patterns in the data coherent with the rest of the themes and allowing the responses that did not fit any of the previous criteria to be added to the matrix. This was done by a repeated reading of the transcripts while comparing every instance with the themes already defined. Some instances are more easily categorised and "lead the way" in building the realm of each theme. After a while, boundaries emerged that were distinct enough to suggest that an utterance probably belonged to a new theme. This suspicion urged me to re-examine all instances to see if one of them in fact fitted the new theme better, and thus, with every new suggested theme, the process repeated itself.

Critique of the qualitative analyses

One criticism of my first study method, reported in article I (Nyman & Sumpter, 2019) and article II (Nyman, in press), is the way of organising the responses according to the questions. It can be argued that this way the researcher predefines the respondent's interpretations and that the connection between motivation and emotion is inherent in the questions. An alternative strategy would have been to approach the data in a purely inductive fashion, drawing every theme directly from the data. However, in humble consideration of my own learning curve and the scope of this study, such a methodology could have proved perilously complex. That said, an interesting alternative using a more purely inductive method could have been phenomenography (e.g., Marton et al., 1997). Such a methodology would probably have produced a more condensed result, with a strong potential for usefulness for further research as well as for practice.

Another potential critique concerns the reliability of the coding process, which included transcribing, coding and stress-testing categories. All these issues, including tentative results, were subjected to discussion with other researchers, in seminars, conferences and meetings. These deliberations generally focused on difficult or ambiguous instances.

Conducting interviews, especially with children, is challenging in every part of the process. In relation to the analysis this is tightly linked to the transcription process, as this is where the individual student becomes a respondent and is where the analysis starts. The preceding pilot study enabled me to device several principles around spelling, pausing and nonverbal communication relevant for the coherence between transcription and analysis.

Quantitative analysis

Items 1, 2, 5 and 7 concern emotion and item 4 motivation, and the analysis was made in two different ways, summarised in Table 2 and Figure 2.

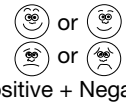
Participants used Likert scales to respond to both items 1 and 2 and they were thus analysed in the same way as either positive or negative when a respondent gave either of the two alternatives, and neutral when the response contained both positive and negative alternatives.

Because items 4–7 asked the respondents to answer in their own words, the analysis of these items must be considered a combination of quantitative and qualitative. The quantitative numerical analysis, which computed sums, percentages and significance levels was based on a qualitative interpretation and coding of responses to the questionnaire items. The first step was deductive and followed the coding in Blomqvist et al. (2012) from whence the instrument came. To allow a certain amount of nuance, the coding was supplemented with a neutral category.

The coding of the emotional content in the images in item 7 (Figure 2) was based on the same generic facial expressions as in the Likert scale faces. A substantial number of the illustrations also contained text and signs, as well as various features common in comic drawings. For example, lightning bolts and skulls were negatively associated, and sunshine, musical notes and light bulbs were positively associated, following this artistic tradition.

Table 2

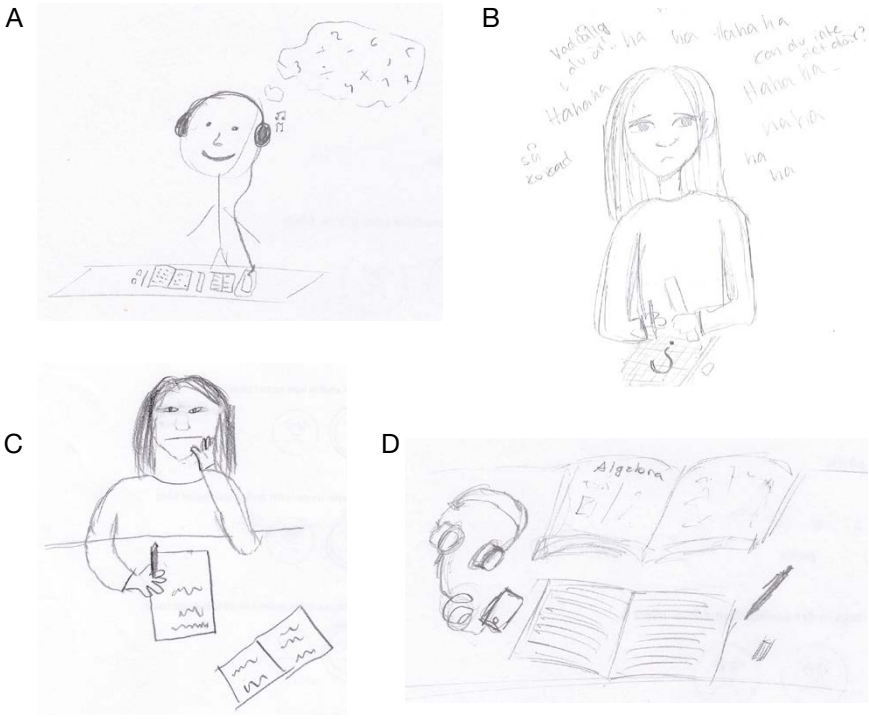
Summary of codes and response examples from Article III. Swedish original within square brackets.

Item	Response type	Codes	Examples
1 & 2	Likert	Positive	
		Negative	
		Neutral	Positive + Negative
4	Free-text	Intrinsic	Because it's fun [För att det är roligt]
		Extrinsic	Because I have to, it's a subject at school [För att jag måste, det är ett ämne i skolan]
		Mixed	Because it's fun and I have to [För att det är kul och jag måste]
		Neutral	To learn more [För att lära mig mer]
5	Free-text	Positive	I feel happy [Känner mig glad]
		Negative	I think it's hard, and boring [Jag tycker det är svårt, och tråkigt]
		Neutral	Sometimes good, sometimes boring [Ibland bra, ibland långtråkigt].

A supplementary, qualitative analysis was made that cross-related items 4 and 5 looking for instances where emotion was a part of the response to a question about motivation, and oppositely, where motivation was a part of the response to a question about emotion. This analysis was made in two steps: first, every response to item 4 that contained any implicit or explicit reference to emotion was marked. Item 5 was treated correspondingly. The marked instances for item 4 were subsequently analysed using the same lens as described above for analysing emotion, and item 5 correspondingly for analysing motivation.

Figure 2

Emotional categories for Item 7. Examples of response images.



Note. Panel A: Positive. Panel B: Negative. Text in image: ”How stupid”; ”You’re so bad!”; ”Don’t you know that?” Panel C: Neutral. Panel D: No emotional information.

After coding, relative frequencies for the different categories were calculated, along with testing for significance. Generally, 0.05 was considered significant. However, in the tables, numbers rounded off to one significant digit are

reported. Data in this study are nominal, therefore χ^2 was chosen for significance testing since the main part of the correlations tested was 2 × 3 or larger (Bryman & Nilsson, 2018). The χ^2 is widely used in the social sciences (Edling & Hedström, 2014). The χ^2 value needs to be related to the degrees of freedom value, that in turn is a function of the number of variables tested; thus the χ^2 test tends to show an increasing measure of significance when the number of variables grow, for example, from a 2 × 3 to a 2 × 4 table. Therefore, whenever a correlation showed significance an attempt was made to reduce the number of variables tested to see if the difference among this reduced number of variables also showed significance. For example, when the different frequencies between the three emotional categories Positive, Neutral and Negative showed significance another calculation was made connecting only the two extremes Positive and Negative.

Summary of Papers I–III

This thesis is based on data from two studies, where papers I and II report results from the first study and paper III reports study two.

The summaries below follow the same format: first a description of each aim and research questions, then one or two sentences on method. Most of the text summarises the results in several points. Under each of these points the results are presented and discussed and then, in conclusion, a sentence or two touching upon implications for practice or further research.

I The issue of "proudliness"; Primary students' motivation towards mathematics

This paper was co-authored together with Lovisa Sumpter, even though, as previously described, I carried out all data collection, transcription and analysis. In these phases Sumpter had a supervising role. Correspondingly, in the subsequent writing process, the main volume of the text was written by me with Sumpter also contributing to the text but primarily in a supervising capacity.

The aim of this paper was to discuss a nuanced understanding of students' expressed motivation and the potential relationships between different affect-related constructs, framed by the research questions:

1) What different types of motivation do primary school students express? and
2) How are these different types of motivation interrelated in the students' responses?

Focusing on motivation, responses in relation only to question four, "Why do you do mathematics?", were chosen and analysed.

The results are summarised in five points, the first being the different motivation types in Table 3.

Table 3 shows that motivation with an extrinsic origin is more common in the data, but also that students express a rich variety of motives for doing mathematics that are in line with previous research but also illustrate further nuances within these constructs.

Table 3

Explicit motivation expressed by students in year 2 and 5. Total number of instances within brackets.

Main Themes	Subscales	Subthemes
Extrinsic motivation	Outward (27)	Important for the future (12)
		To be able to calculate things (8)
		To get/manage a job (6)
		To produce an "answer" (explicit) (1)
	Compensation (5)	Not to make calculation errors (3)
		You succeed if you make an effort (2)
Intrinsic motivation	Cognitive (15)	To learn (13)
		Want to try new things (1)
		Maths makes you better (1)
	Emotional (9)	Fun/I like it (7)
		Exciting (1)
		No stress (double negative) (1)

Note. From "The issue of 'proudliness': Primary students' motivation towards mathematics," by M. Nyman, and L. Sumpter, 2019, *LUMAT: International Journal on Math, Science and Technology Education, 7*(2), 80–96–80–96. https://doi.org/10.31129/LUMAT.7.2.331

Future needs are considered important, but often with vague references to being able to conduct various professions or to just manage everyday life, for example, buying groceries.

The reason for doing mathematics is often expressed as "to learn". In some cases, this has an extrinsic origin, but it can also be intrinsic. This intrinsic origin seems to be personally or normatively loaded, giving rise to two different subthemes: Intrinsic motivation+Cognitive+Learn – Personal type (ICL-P) and Intrinsic motivation+Cognitive+Learn – Normative type (ICL-N), see Figure 3.

An interplay between motivation and emotion and other factors is found in the data. The excerpt from the interview with Casper (grade 2) illustrates this:

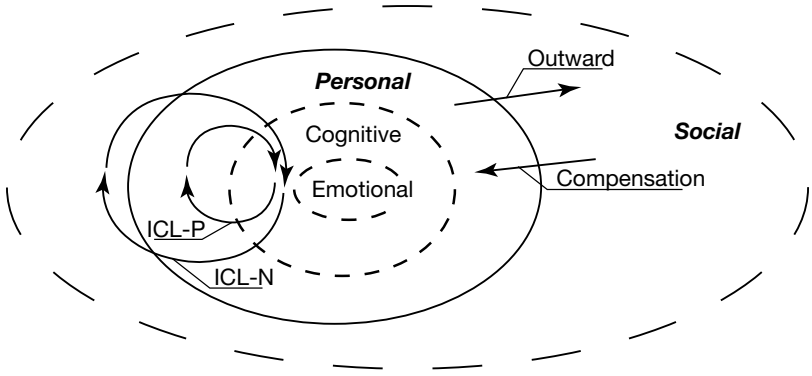
> Interviewer: Why do you do maths?
> Casper: Why I do maths?
> Interviewer: Mm?
> Casper: Because I like it very much… and it just feels good when I do maths.
> Interviewer: Aha, does it?
> Casper: Mm …
> Interviewer: Can you try to describe that feeling of "good"?
> Casper: It feels warming. Proudliness [Swedish: "stoltlighet"]. And it feels like you're going to make it.

In this passage Casper expresses being motivated by his emotions, thus indicating an interplay of some kind.

Lastly, Figure 3 shows a model that attempts to connect the different motivational constructs and illustrate their role in connecting the person to the social world.

Figure 3

Model of relationship between constructs.



Note. From "The issue of 'proudliness': Primary students' motivation towards mathematics," by M. Nyman, and L. Sumpter, 2019, *LUMAT: International Journal on Math, Science and Technology Education, 7*(2), 80–96–80–96. https://doi.org/10.31129/LUMAT.7.2.331

The article discusses both the interplay between motivation and other affective constructs – primarily emotion, and the intraplay between dimensions within the motivational dimension. The result proposes that the Intrinsic motivation subscale Cognitive consists of subscales; one of them being the Double negative subscale (where students' express intrinsic motivation due to positive feelings of non-stress), and another the To Learn subscale. To learn is in turn split into Personal and Normative type. Figure 3 illustrates the levels at which these different motivational constructs play out. Outward and Compensation, two one-directional constructs, connect the inner world to the social exterior, and ICL-P and ICL-N can be seen as feedback loops, loading motivation based on cognitive factors with personal or normative preferences. The article discusses the need for re-evaluating the role of motivation in educational research as well as in teaching and learning: in education the view on extrinsic levels of motivation need to be discussed and the canonical and one-sided educational push for enthusiasm-driven intrinsic motivation needs to be challenged.

II Primary students' expressed emotions towards mathematics

This second paper aims to explore student emotions towards mathematics education, particularly nuances in their expressed emotions. The two research questions were:

1) How do students express emotion in relation to mathematics and what are the characteristics within these expressed emotions?
2) How and by what mechanisms are students' emotions linked to expressed motives?

The data come from the same interviews as in paper I, but here the analysis focuses on responses to the three items explicitly framing emotion: "What do you think about maths?", "(h)ow do you feel before a maths class?", and "(h)ow do you feel when you do maths?"

One result is Table 4: a mapping of the eight basic emotions described by Lövheim (2012) that was the basis for the deductive analysis, against the expressions present in the data. Each one of the expressed emotions are exemplified with an excerpt. The analysis of the "mismatches" between the theoretical constructs and the findings in the data is further described below.

Table 4

Excerpts and themes mapped against constructs from theory of Basic emotions

Excerpt	Expressed emotions	Basic emotions
It's fun and exciting	Joy	Excitement
It feels like you're on top of things	Contentment	Joy
It feels safe having a kind teacher that helps you if you need	Relief	—
—	—	Surprise
Sometimes it's a bit tiresome	Discontentment	Distress
You get stressed about whether it's correct or not	Stress	Fear
I'm ashamed to ask	Shame	Shame
—	—	Disgust
—	—	Anger

Note. Basic emotions see Lövheim (2012). From "Primary students' expressed emotions towards mathematics," by M. Nyman, in press, *Sustainable mathematics education in a digitalized world. Proceedings of MADIF 12. SMDF.*

In relation to RQ 1, Table 5 contains a summary of the ways in which students express emotions towards mathematics. Both positively and negatively loaded emotions are represented (Lövheim, 2012): joy, excitement, shame, and distress. However, fear and distress manifests more like stress and discontentment,

respectively. These instances appear in various situations during lessons, and link to personal or social dimensions (e.g., Karlsson, 2019; Samuelsson, 2011). The lack of explicit instances of anger, fear or disgust raises the question of whether these emotions are "hidden" and expressed as stress; a suspicion which calls for further research.

Table 5

Students' expressed motivations for emotions experienced in relation to mathematics education. Number of instances within brackets.

EMOTION	Cognitive	Motivational	Technical	Personal (relates to self)	Social
POSITIVE (91)					
Joy (34)	Process (7) Position (8)	Challenge (19)			
Contentment (15)			Situation (14)	Autonomy (1)	
Relief (20)			Control (20)		
NEGATIVE (64)					
Discontentment (1)		Boredom (1)			
Stress (52)			LC Temporal (7)	LC Personal (45)	
Shame (11)				Personality (8)	Social (3)

Note. LC = Lack of Control. From "Primary students' expressed emotions towards mathematics," by M. Nyman, in press, *Sustainable mathematics education in a digitalized world. Proceedings of MADIF 12. SMDF.*

Looking at the sums of positive emotions, we see that the total number of positive instances (91) is substantially higher (32%) than the sum of the subsets (69) compared to the negative emotions where all negatively loaded responses are motivated. This reflects a socio-cultural difference in how positive versus negative emotions and moods are communicated.

The different ways students justify emotions (Table 5) can be understood as a starting point for understanding the mechanisms connecting emotion to other affective concepts. Thus, Table 5 is an attempt at answering RQ 2. For example, it implies an interplay between motivation and emotion, also reported by Nyman and Sumpter (2019) (i.e., Paper I), and Hannula (2012). Moreover, the Personal dimension appears to be very similar to Samuelsson's "self-concept" (2011). However, the Technical factors do not seem to be part of any previous theory. Further research – preferably using a participatory methodology – could perhaps capture the fleeting emotion-connected dimensions that occur momentarily to deepen the understanding of affective mechanisms.

The article also discusses the theoretical strength of viewing teaching through the lens of activity theory (e.g., Arievitch, 2017; Radford, 2015). The way it connects (student) action to motives and emotion places the power to influence students in the hands of the teacher through a culturally rooted activity-based teaching built on inherent student needs. Finding and describing design principles for this type of teaching should also be the subject of further research.

III Secondary students' expressed emotion and motivation towards mathematics

The third paper discusses potential differences in expressed affective experiences related to mathematics between age groups (students in grade 8 and 9, aged 14 and 15, respectively) and gender. This paper is framed by the following research questions.

1) How do students express their feelings towards doing mathematics?
2) How do students express their motivation for doing mathematics?
3) How do motivation and emotion interrelate?
4) What gender differences in relation to questions 1 and 2 can be found?

The analysis was primarily quantitative, with data coming from 222 questionnaires (110 girls, 109 boys and 3 who did not choose a gender) completed by students in grade 8 (n = 78) and grade 9 (n = 144).

The results are summarised here in four tables and one concluding figure. The first, Table 6 shows the distribution of responses for the items focusing on emotion.

Table 6

Distribution of responses over the Emotional subscales in relation to item 2 ("How do you feel before a maths lesson?"), 5 ("How do you feel when you do maths?") and 7 ("Draw a picture of yourself when you do maths"). % (n).

Item	Respondent	Positive	Negative	Neutral	Total	p
2	Grade 8	55 (43)	37 (29)	8 (6)	100 (78)	
	Grade 9	55 (79)	44 (63)	1 (2)	100 (144)	> 0.5
	Girls	54 (59)	43 (47)	4 (4)	100 (110)	
	Boys	57 (62)	39 (43)	4 (4)	100 (109)	> 0.5
5	Grade 8	56 (44)	29 (23)	14 (11)	100 (78)	
	Grade 9	52 (75)	40 (57)	8 (12)	100 (144)	> 0.1
	Girls	53 (58)	42 (46)	5 (6)	100 (110)	
	Boys	54 (59)	30 (33)	16 (17)	100 (109)	< 0.05
7	Grade 8	54 (25)	15 (7)	30 (14)	59 (46)	
	Grade 9	48 (44)	32 (29)	20 (18)	63 (91)	> 0.05
	Girls	54 (38)	25 (18)	21 (15)	65 (71)	
	Boys	48 (30)	27 (17)	25 (16)	58 (63)	> 0.5

Note. Significances computed for grades and gender separately.

We see that most of the students reported positive emotions towards mathematics and that no significant differences are found between ages. Differences between genders are not salient either; only in item 5 is the difference between girls and boys significant and this due to the difference in negative and neutral responses. The positive scores are almost identical.

Looking at motivation in Table 7, we see the results showing that a majority of the students are extrinsically motivated.

Table 7

Distribution of responses over the Motivational subscales in relation to item 4. % (n).

Respondent	Intrinsic	Mixed	Extrinsic	Neutral	Total	p
Grade 8	29 (23)	6 (5)	63 (49)	1 (1)	100 (78)	
Grade 9	22 (31)	8 (12)	66 (95)	4 (6)	100 (144)	> 0.1
Girls	18 (20)	12 (13)	70 (77)	0 (0)	100 (110)	
Boys	29 (32)	4 (4)	61 (66)	6 (7)	100 (109)	< 0.005

Note. Significances computed for grades and gender separately.

The differences between ages seen in Table 7 are not significant; however, a significantly higher number of girls report being extrinsically motivated or express a mixture of intrinsic and extrinsic motivation than boys do. This is in line with what has been previously reported in, for example, Giota (2013), who talks about girls having a more social and responsible rationale for motives.

Reports in previous studies, as well as in my papers I and II, that emotion and motivation are interrelated, resulted in a search for expression of emotional valence in relation to questions about motivation. This is illustrated in Table 8.

Table 8

Types of relation between Motivation category and Emotional valence in Motivation responses

Example from Item 4	Motivation category	Emotional valence
”Mostly I think it’s fun an challenging, so I just keep on /.../. ”	Intrinsic	Positive
”Because it’s fun and I have to.”	Mixed	Positive

Note. Item 4 = ”Why do you do maths?”

A similar search was made for expressions of motivational connection in relation to questions about emotion. This is illustrated in Table 9.

Table 9

Types of relation between Emotion category and Motivational connection in Emotion responses.

Example from Item 5	Emotion category	Motivational connection
”Not fun, but necessary.”	Negative	Extrinsic
”It’s important /.../.”	Neutral	Extrinsic
”Pretty motivated when I’m doing well.”	Positive	Extrinsic
”If it’s too easy it’s boring /.../ challenging tasks makes me focused.”	Positive	Instrinsic

Note. Item 5 = ”How do you feel when you do maths?”

Only a subset of the responses contained information about these connections; the results therefore shown in tables 8 and 9 are based on a qualitative analysis which aimed to establish the connections. In this set of data, no other type of connection was present. Combining tables 8 and 9 made it possible to construct the dependence model shown in Figure 4.

Figure 4

Model of dependence between Motivational and Emotional dimensions

Motivation ⟷	Emotion
Intrinsic	Positive
Mixed	Positive
Extrinsic	Positive Neutral Negative

In light of the relationships shown in Figure 4, Article III discusses the inter-relation between motivation and emotion and suggests a re-evaluation of the thinking behind pressurising educators to make mathematics education intrinsically motivating.

Discussion

The purpose of this thesis is to contribute to a more nuanced understanding of students' motivation and emotions in relation to mathematics and mathematics education, by tapping into the experiences of students of different ages, across all stages of compulsory schooling. In this section, I summarise and discuss these contributions in five distinct paragraphs, suggesting avenues for further research and pointing to some implications for practice.

Can motivation be separated from emotion?

The straightforward answer is "yes, of course we can". Simply by using the definitions presented earlier we see a substantial difference between the two: motivation is about a person's driving force for action and emotion is the person's more or less conscious feelings in response to stimuli. However, in educational situations this distinction quickly starts to appear hazy and difficult to pinpoint. For example, it is not uncommon that students consider motivation to be an emotion, like Ellen saying, "I feel motivated" or a bit less directly, like Eric saying, "I feel good when I'm motivated".

Previous research has acknowledged this complexity between the concepts, and Hannula's solution (2012) was to include both (with cognition) in the three dimensions of the larger concept affect. But capturing the dynamic movement of educational and learning situations requires an understanding of the parts in motion. And perhaps this structure is not as stable as Hannula suggests, perhaps they cannot productively be seen as separate concepts at all.

In addition to the examples above, showing that students can very well consider emotion and motivation inseparable, we can look at the results presented in Table 5 (Article II). Involuntarily hidden among the numbers are the 22 instances of responses loaded with "purely" positive emotions. Bearing in mind that this table captures responses to the question, "(h)ow do you feel when you do maths?", these 22 responses were simply, "good." More important is the fact that the remaining 135 instances, including all of the emotionally negative ones, communicate an emotion in terms of motivation.

Another feature of Table 5 are the 87 instances that in various ways refer to control. One way to understand this is through the control-value theory (Pekrun, 2006) that sees control as an expectation about one's ability to perform. This belief-based way of looking at control resonates well with the categories LCP and LCT (Lack of control, Personal and Temporal, respectively). Here the

respondents typically voice having low expectations of their ability to perform well. These expectations are based on previous experiences and thus, linked to self-efficacy beliefs that have developed over time. We can see the negative emotions of stress as an effect of low motivation due to the low expectations that the student has developed over time. But we can also sense the feedbacking effect of negative emotions on the student's motivation, just as Pekrun (2006) says, "(e)motions can induce and modulate students' interest and motivation to learn" (p. 326). Below, two fifth-graders respond to the question "How do you feel when you do maths?" (some phrases have been highlighted for discussion purposes):

> Frida: I feel okay, pretty *content* and when I work in the book I <u>always finish on time</u>.
> Interviewer: What is it that makes you content?
> Frida: That *I learn* that I must *take my time*. Not to be stressed, that's when I <u>get it wrong</u> and so on. I mustn't work too fast.

> Vera: When I do maths, sometimes, as I said, I feel stressed but sometimes I feel pretty calm. It <u>depends on where I am [in the workbook] and how it is</u>.
> Interviewer: Does it depend only on where you are, or can it be affected by something else as well?
> Vera: Well it can also be because I feel pretty stressed when *we have this, as I said, this work plan*, so I can feel stressed. So that affects it also.

The underlined phrases can be seen as instances of control as in the belief-based way described by Pekrun (2006); Frida knows that she "always finish[es] on time" and that she "get[s] it wrong" when she is stressed. Frida expects, based on previous experiences, to perform in a satisfactory way and this makes her happy. However, the same mechanism seems to have the inverse effect on Vera. She is not certain that she will be able to perform in a satisfactory way; her previous experiences tell her that this rather depends on where she is in the workbook, thus placing the power to decide outside herself. And the emotional effect on Vera is that she feels unstable and runs on a stress–calm continuum. Looking at Frida and Vera this way focuses on their performance and the emotions and motivation connected to that, not unlike the beliefs about expectation discussed by Sumpter (2013): "the problem arises when expectations become the dominating factor compared to mathematical knowledge" (p. 1131). However, in the example with Frida and Vera the emotional dimension is highlighted as a potential factor in the formation of the belief.

Another way to understand this expressed dimension of control is through Ryan and Deci's (2000) motivation framework based on autonomy and competence (alongside relatedness). The autonomy-dimension is most interesting here, which is why I italicise this for emphasis. Looking at these statements helps us to understand that both girls express some dimension of autonomy.

On the one hand, Frida is "content" and feels that she "learns". On the other hand, it seems that the fleeting uncertainty in Vera's feelings seem connected to her low sense of autonomy.

Both girls actually report similar situations where they are confronted with external tasks that seem to contain no inherent motivational qualities; these qualities come solely from producing correct answers and on time. They differ in how they tackle the situation, and this, in turn, is affected by their previous experience of similar situations; again, the formation of motivational beliefs and the connection to goals discussed by Sumpter (2013) can be recognised. Thus, the results of this study indicate that problems arise for students whose need to feel autonomous is not fulfilled, and that this in turn can be expected to influence their beliefs. This potentially risks that some students develop a pattern of avoidance (Trezise & Reeve, 2017). It would indeed be interesting, both for research and educational purposes, to see what type of emotion and motivation a student who is familiar with a different (and differentiated) type of mathematical task would describe.

Strong motivation for mathematics

From a qualitative standpoint, students express a variety of different types of motivation for doing mathematics (as can be seen in Table 1, Nyman & Sumpter, 2019) – both intrinsic and extrinsic (Ryan & Deci, 2000); the extrinsic subscale Outward (Amabile et al, 1994) is prominent in the data and is further divided into subthemes that reveal the students' ultimate aims for engaging in mathematics. Already, the youngest respondents are considering mathematics to be of great importance in the future, like Louise in grade 2:

> I should learn so if I work at the till when I'm grown up for instance and there is a fruit that cost 20 crowns, and the person gives perhaps 40 crowns, then I should be able to calculate how much he/she should get in return.

Also, the two intrinsic subscales, Cognitive and Emotional (Sumpter, 2013) are further divided into subthemes that concern wanting to learn and finding mathematics joyful and exciting. Respondents generally express a strong motivational drive for doing mathematics.

Looking deeper into the category To Learn (a subtheme under the intrinsic subscale Cognitive), the analysis suggests an even finer division, with a personal or normative connection, respectively – to simplify, a division between "wanting" and "needing", two words frequently used by respondents when talking about why they learn mathematics. Looking at detail, the proposed construct, ICL-N (Intrinsic motivation+Cognitive+Learn – Normative type), illustrates that intrinsic motivation can have traces of external impact, thus challenging the

view of motivation as a linear construct (e.g., Ryan & Deci, 2000) and pointing to additional research being needed if we want to better understand motivational factors in affective mechanisms.

As recently mentioned, students' motivation for doing mathematics seems generally strong. When adding quantitative data from the questionnaires administered in grades 8 and 9, we see that the motivational drive is strong here as well: only 11 percent of students express neutral or mixed motivation, and the remaining 89 percent report being intrinsically or extrinsically motivated. Still, it is not uncommon to hear teachers lamenting that their students are "unmotivated". Maybe this is due to teachers confusing *un*-motivated and extrinsically motivated. Indeed, Table 2 in Paper III shows that to a large extent the students (about 65%) are extrinsically motivated, compared to 24 percent reporting to be intrinsically motivated. However, educational research from different fields (e.g., Young, 2015; Dweck, 1988; Schoenfeld, 2020) shows that external expectations and demands are needed for a person to gain knowledge beyond what Biesta calls "the context dependent" sphere. One implication could be that teachers acknowledge their students' conviction that mathematics is important by letting them engage with theoretically challenging content, and by helping them to see the strength of their extrinsic motivation.

Boredom – a complex concept

The dimension of boredom is touched upon in Article II as more complex and potentially more detrimental for student learning than a dictionary definition implies. Analysing the questionnaire responses from grades 8 and 9 shows that almost 20 percent of students (40/222) explicitly mention boredom in response to the question, "(h)ow do you feel when you do maths?" But more importantly, the responses are of a very different type. One type is close to a dictionary definition, like Karen (grade 8) saying: "A lot of repetition of stuff I learnt a long time ago, it's BORING". Karen's use of the word implies a very low level of motivation due to her experiencing low levels of challenge, enjoyment or relevance in the way the subject is taught. That students experience mathematics this way is problematic and is in part what Lewis (2013) means by "disaffection": "low engagement and perceived lack of relevance" (p. 71). However, the two ninth-graders Dennis and Blenda, appear to be saying something different, respectively: "It's fun when you understand but can be boring during lessons when you don't understand"; "It's boring and confusing sometimes".

Both examples reveal dimensions of boredom connected to knowledge and academic self-concept (Samuelsson, 2011), and there are instances in the data that potentially hide a range of different emotions, such as anger, frustration, shame and hopelessness, under the expressions "boring", "bored" or "tired".

Lewis (2013) talks about students being affected by several affectively loaded factors, from within as well as outside school. This implies that educators need to exercise caution when assessing the emotional state of a student who claims to be "bored", especially given students' emotional and motivational fragility when they are feeling negative or disaffected (Lewis, 2013). Further research, directed at unveiling this hidden meaning, is also needed, especially in light of Schukajlow et al's (2017) reporting an increasing level of boredom from mid-primary to the end of compulsory schooling.

As a general remark in relation to the discussions above, it must be noted that the quality of the "qualitative" results from grades 8 and 9 is limited since they are based on questionnaire responses. It is reasonable to assume that the nuances would have been richer if I had posed follow up-questions in a typical interview manner. This assumption also calls for follow-up research that targets these nuances.

Boys and girls have different experiences concerning emotion and motivation

Previous research both reports the existence of gender differences relating to emotions (e.g., Rodriguez et al., 2020), and finds small or no significant differences between genders (e.g., Yavuz, 2018). A possible explanation is that gender differences are primarily the effect of gender stereotypes and expectations about gender (Alawi & Mubarak, 2019 cf. McGeown & Warhurst, 2020). The results of this study indicate that boys and girls experience positive emotions towards mathematics almost identically. But boys significantly more often express neutral emotions towards mathematics, and girls significantly more often express negative emotions towards mathematics. Thus, while a difference does exist between the genders, the difference is not consistent over the whole emotional spectrum, which makes the stereotype-expectation theory difficult to accept.

The explanation for the gendered differences in the non-positive responses probably lies elsewhere, perhaps more in relation to beliefs (like self-efficacy and self-concept) and motivation towards mathematics. Since this study did not primarily focus on issues of gender, further research is needed to explain these differences.

Looking at motivation, there is the same lack of consensus about gender differences in previous research as there is in relation to emotion (e.g., Davidovitch & Yavich, 2018, cf. Rodriguez et al., 2020). The results of this study, however, indicate that boys more often than girls express intrinsic and neutral motivation towards mathematics, and girls more often than boys express extrinsic and mixed motivation towards mathematics. Given that the respondents are at an age when girls are generally more mature than boys of the same age, the

girls group can be expected to adopt a considerably more adult view of life than the boys. Two elements of such an adult view are being more responsible and thinking more about the future. This possibly explains why the girls report being more extrinsically motivated towards mathematics; they simply know that it is something they have to do at school, and they need both grades and knowledge in mathematics for further studies. Elisabeth in grade 9 is a good example: "Because you want to do your best for the grades, and it's important to know maths in life."

As previously discussed, the general connection between emotion and motivation is strong but the causal mechanisms are still uncertain (e.g., Frid et al., 2020) and further research is required. One example being, as can be seen as one result of this study, that since the proportion of girls with positive emotions towards mathematics is equal to the proportion of boys implies that an extrinsic drive does not automatically result in negative emotions. Generally, it seems that more detailed knowledge concerning gender issues is needed to understand the cause, and more importantly, the effect of girls' and boys' different motivations towards mathematics – one effect being the gender asymmetry in young peoples' choosing of STEM subjects in higher education (see, for example, Stoet & Geary (2018). Thus, it seems that in addition to more research, new research approaches need to be adopted to reach a consensual understanding of the gendered dimension in the mathematics-related affective domain.

Concluding thoughts and ideas for practice

Considering Young's (2015) division of learning into context-dependent and context-independent, it is reasonable to assume that the higher up you look in the education system the more context-independent the learning will be.

Thus, for mathematics education to extend beyond everyday functionalism and a mundane societal usefulness it will inevitably need to contain elements of struggle. And this translates into a challenge for educators to balance students' need to feel safe when doing schoolwork with activities that have the potential to enable context-independent learning.

The concept of boredom, discussed here as being more complex and detrimental to students and learning than the word's purely lexical meaning suggests, is a challenge for the teaching profession. That reports of boredom are so frequent in the data must be considered a problem; it is never desirable for an activity as delicate as learning to be considered boring. However, far more problematic is when it turns out that the word in most cases is not used in its common lexical sense but rather to conceal the sentiment that "I don't understand but I don't dare to tell". As a mathematics teacher myself, I have on numerous occasions heard students complain about being bored, and I now cannot help suspecting

that in many cases what the students actually meant was something far more worrying than just wanting "some more action".

The connection and interplay between motivation and emotion exist, so it is therefore important to think of these concepts together. In addition, while these concepts are internal to the individual, they are also interacting with social dimensions. It is, therefore, not possible for teachers or educators to "create" motivation or positive emotions in the students. Rather, teachers need to create conditions, through tasks and activities, that students find meaningful and want to engage with. If the tasks and activities are designed to engender in students a need to solve them and the motivation to do so, then intrinsic motivation will arise and the consequent emotions will be related to solving the task or completing the activity. And even though the development of such tasks and activities is not a simple undertaking, through systematic and investigative approaches teachers can do this.

Thus, the results suggest that emotion and motivation, through their reciprocal interconnections, present educators with a complex challenge – facing the unique emotion-motivation pattern in every student. Even though it is probably impossible to map every aspect of this complexity, my results indicate that a common feature when students express negative emotions and low motivation is their seeming lack of having experienced ownership of the power of mathematics. The individual's emotions and motivation come from their experiences. We must, therefore, assume that we as educators can play a part in transforming students' negative feelings into a more positive relationship with mathematics, thus enabling them to claim ownership of mathematics practices and powers, and to see that mathematics is what Schoenfeld (2020) describes as a discipline of exploration and sense making.

Sammanfattning på svenska

Vad tycker elever om matematik? Grundskoleelevers känslor och motivation till matematik

> ”Affekt utgör en centralpunkt för inlärning, likställd med kognition” (Lewis, 2013, s. 74, min översättning)

Med utgångspunkt i Lewis’ konstaterande kan vi anta att frågor relaterade till affekt spelar en viktig roll för elevers kunskapsutveckling och lärande. Den här uppsatsen fokuserar motivation och känslor, och består av två delstudier, den första primärt kvalitativ, den andra primärt kvantitativ, som utförts på elever från grundskolans tre stadier.

Syftet med arbetet har varit att söka efter nyanser i hur elever från olika stadier i grundskolan beskriver sina känslor till och motivation för matematik.

Teoretiskt har det här arbetet vägletts av Hannulas metateori om affekt som han delar upp i motivation, känslor och kognition (se t.ex. Hannula, 2012). Var och en av dessa tre delar består av en kombination av fysiologiska, psykologiska och sociala dimensioner som utspelar sig i såväl momentana som mer kumulativa tidsplan.

Motivation förstås i det här arbetet som drivkraften som får oss att göra saker och ting för att åstadkomma något visst, önskat resultat. Schunk m.fl. (2010) definierar motivation som ”den process varigenom målorienterade aktiviteter initieras och upprätthålls” (s. 4, min översättning). Viktigt att komma ihåg är att motivation kan vara såväl positiv som negativ, styrt av personens mål att antingen uppnå eller undvika något. Motivationen kan antingen ha ett inifrån kommande ursprung kopplat till personens önskningar och lust, eller ett utifrån kommande ursprung med målet att lösa externt uppkomna behov och krav. De här två formerna kallas för intrinsisk respektive extrinsisk motivation (se t.ex. Ryan & Deci, 2000). För att undvika anglicismen används på svenska ofta istället begreppen inre och yttre motivation (se t.ex. Anna Erlandssons översättning av Skaalvik & Skaalvik, ”Motivation och lärande”, Natur & Kultur, 2016), och jag kommer härifrån använda de begreppen. Man kan se motivation som ett slags spektrum; från amotivation, där individen helt saknar motivation, via yttre motivation till allt högre grader av inre motivation (Ryan & Deci, 2000). Den beskrivningen gör det dock svårt att se att individen kan uppleva en blandning av inre och yttre motivation. Ett mer tvådimensionellt synsätt där de två versionerna existerar oberoende av varandra möjliggör att individen kan uppleva en blandning av båda typerna (Amabile m.fl., 1994) och vardera av de två olika typerna av motivation kan då ses sönderfalla i flera underkategorier (Amabile m.fl., 1994; Sumpter, 2012).

Känslor är det här arbetets andra huvudfokus. Damasio (2004) beskriver känslornas neurologiska mekanism, från någon form av stimuli till att fullt ut känna känslan, i fem steg. Det första steget inbegriper själva initieringen av känslan som kan härröra från såväl inifrån som utifrån kommande stimuli. Det engelska språkets tillgång till orden *emotions* och *feelings* möjliggör här dessutom en funktionellt viktig distinktion av medvetandegraden mellan dessa två begrepp: det som individen ”känner” under steg ett kallar Damasio för en emotion. De följande två stegen beskriver olika responsmönster i form av överföring av olika aktiva substanser i kroppen och nervsystemet. De två sista stegen beskriver den process där den ursprungligen omedvetna känslan – alltså det Damasio kallar emotion – blir medveten för individen och individen känner känslan. Med den bakgrunden definieras känslor, i det här arbetet, som ”medvetna eller omedvetna mentala tillstånd, initierade av betingelser som vi bedömer som relevanta för våra behov och som motiverar beteenden att uppfylla dessa behov” (Shirmer, 2015, s. 26, min översättning).

Ett flertal studier belyser att elevers (och även vuxnas) affektiva relation till matematik är olika för pojkar och flickor (t.ex. Alawi & Mubarak, 2019; Frid m.fl., 2020; McGeown & Warhurst, 2020; Sumpter, 2012). Det är därför relevant att studera skillnader relaterade till gender (Nielsen, 2017), bland annat för att skolan är en plats där gender ”görs” (Bjerrum Nielsen, 2003; Wedege, 2011), dvs. en socialt och kulturellt betingad situation som påverkar individens syn på sig själv.

Resultaten som presenteras i den här uppsatsen kommer från två studier. Studie ett är en intervjustudie där 19 elever i årskurs 2 och 5 intervjuats, och studie två en enkätstudie med 222 enkätsvar från elever i årskurs 8 och 9. (Studie två innehöll även) en intervjudel med 24 elevintervjuer med elever i åk. 8 och 9, men av tidsskäl behandlas inte detta data i den här uppsatsen.)

Som intervjuguide användes en enkät (se appendix D) som tidigare använts av Dahlgren Johansson & Sumpter (2010) och Blomqvist m.fl. (2012), också de i årskurs 2 och 5 men med kvantitativa analysmetoder. För att möjliggöra vissa jämförelser gjordes valet för min studie att fokusera samma årskurser. I min andra delstudie användes återigen samma enkätverktyg men denna gång just som enkät som genomfördes av fem klasser i årskurs 8 och sex klasser i årskurs 9.

Resultaten från studie ett redovisas i två artiklar, den ena med fokus på motivation: Nyman & Sumpter (2019), den andra med fokus på känslor: Nyman (in press). Resultaten från studie två redovisas i den här uppsatsen i artikel III som är under arbete.

I Artikel I (Nyman & Sumpter, 2019) framkommer att elevers uttryckta motivation för matematik ofta är vaga referenser till att klara ett ännu avlägset vuxet vardagsliv. Till exempel uttryckt som, ”att lära sig”. Med utgångspunkt i Sumpter (2012) som visat att inre motivation sönderfaller i underkategorier, en kognitiv och en emotionell, tyder analysen av studie ett på att den kognitiva dimensionen i sin tur består av ytterligare mindre teman, en av dem är just, ”att

lära sig”. Vidare uttrycker respondenterna både personliga och normativa skäl för motivet att lära sig. Dessa båda kategorier utgör var sin sammankopplande struktur, den ena till individens självbild, den andra till omvärlden via socialt burna förväntningar. Interaktioner mellan motivation och känslor finns också i data, exemplifierat av Casper som uttrycker att han blir motiverad av sina känslor, men också av den s.k. ”dubbelt negativa” kategorin där elever uttrycker motivation på den emotionella grundvalen av att inte känna sig stressade. Generellt indikerar resultatet att såväl yttre som inre motivation sönderfaller i sex finare underkategorier vardera.

Artikel II (Nyman, i tryck) fokuserar elevers uttryckta känslor till matematik och resultaten visar att elever uttrycker både negativa och positiva känslor till matematik men att de negativa känslorna genomgående åtföljs av någon form av motivering eller exemplifiering av vad som orsakat denna. Jämfört med de grundläggande känslolägen som definieras av Lövheim (2012) finns inte i data några referenser till de starkt negativt laddade varianterna avsky och ilska, däremot såväl oro som skam. Detta kan ses som en indikation på att elever kopplar sina negativa känslor till sig själva och uttrycker skam – exempelvis inför att inte klara en uppgift – snarare än ilska gentemot uppgiften som sådan eller läraren som delat ut den. Den här misstanken antyder ett behov av ett fördjupat utforskande. Vidare visar resultaten på samband mellan känslor och andra affekt-relaterade koncept som motivation och sociala faktorer, men också på mer tekniska aspekter av situationen runt arbetet, exempelvis att få sitta bredvid en kompis eller lyssna på musik. Även dessa aspekter tycks kunna framkalla både positiva och negativa känslor.

På grundval av sambanden mellan motivation och känslor, och med teoretisk utgångspunkt i aktivitetsteorins (t.ex. Arievitch, 2017; Radford, 2015) beskrivning av sambandet mellan behov–motiv–aktivitet, diskuteras avslutningsvis behovet av ett fortsatt utforskande av hur en mer aktivitetsbaserad undervisning, som tar sin utgångspunkt i elevers behov, kan designas.

Artikel III, som är under arbete, är en kvantitativ studie om elevers känslor och motivation till matematik, genomförd på 11 klasser i årskurs 8 och 9. Resultaten visar genomgående inga signifikanta skillnader mellan årskurserna, och få skillnader också mellan pojkar och flickor. Den enda signifikanta skillnaden är att flickor uppger att de i högre grad än pojkarna drivs av yttre motivation. Data indikerar en koppling mellan motivation och känslor också i artikel III. Inre och blandad motivation tycks kopplat endast till positiva känslor, medan yttre motivation koppar till såväl positiva som negativa, men också till neutrala känslor.

I kappan diskuteras den sammanvägda bilden från de tre artiklarna, inledningsvis utifrån frågeställningen om motivation över huvud taget kan separeras från känslor? Det här arbetet antyder att trots att de två fenomenen har tydliga definitioner dyker den ena påfallande ofta upp som en förklaring för den andra – i båda riktningarna, när elever beskriver sin relation till matematik. För att

fördjupa kunskaperna om mekanismerna bakom de här sambanden, och framförallt effekterna på elevers inlärning, krävs sannolikt ytterligare forskning. Några förslag på hur det skulle kunna se ut ges i den här uppsatsen. Ett är nyanserna bakom elevers val av ordet ”tråkigt”. Det tycks användas till en relativt liten del i sin lexikala betydelse och betydligt mer som ett samlingsbegrepp för allt från att man ”inte fattar nåt!” till ”jag är rädd för att min lärare blir arg och tycker att jag är dum om jag frågar”.

Vidare visar data i det här arbetet att elever generellt har stark motivation för matematik, den kan dock vara såväl inre som yttre. De kopplingar som finns mellan motivation och känslor antyder att man i undervisningssammanhang behöver acceptera båda motivationsformerna. Skillnaderna mellan pojkar och flickor är dock ett område där motivationen tycks resultera i ett bortfall av flickor från matematikintensiv högre utbildning (Stoet & Geary, 2018). I ljuset av att flickorna, i det här arbetet, visat sig vara mer yttre motiverade än pojkarna pekar det mot ett behov av att fortsätta undersöka den motivationsformens eventuellt negativa inverkan på individuella val.

References

Alawi, W. S. S., & Mubarak, M. M. A. (2019). Gender Gap in Science, Technology, Engineering and Mathematics (STEM): Barriers and Solutions. *International Journal of Economics and Financial Issues*, *9*(6), 225–231.

Amabile, T. M., Tighe, E. M., Hill, K. G., & Hennessey, B. A. (1994). The Work Preference Inventory: Assessing Intrinsic and Extrinsic Motivational Orientations. *Journal of Personality & Social Psychology*, *66*(5), 950–967.

Arievitch, I. M. (2017). *Beyond the brain: An agentive activity perspective on mind, development, and learning*. Sense Publishers.

Baten, E., & Desoete, A. (2019). Metacognition and motivation in school-aged children with and without mathematical learning disabilities in Flanders. *ZDM*, *51*(4), 679–689. https://doi.org/10.1007/s11858-018-01024-6

Biesta, G. (2011). *Good education in an age of measurement: Ethics, politics, democracy.* Paradigm Publishers.

Bjerrum Nielsen, H. (2003). One of the boys? Doing gender in Scouting. *World Organization of the Scout Movement*.

Blomqvist, A., Elamari, U., & Sumpter, L. (2012). *Grade 2 and grade 5 students' conceptions about mathematics and mathematics education*. 187–196.

Bornemark, J. (2018). *Det omätbaras renässans: En uppgörelse med pedanternas världsherravälde* (1st ed.). Stockholm: Volante.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Bryman, A., & Nilsson, B. (2018). *Samhällsvetenskapliga metoder*. Liber.

Dahlgren Johansson, A., & Sumpter, L. (2010). *Children's conceptions about mathematics and mathematics education.* MaVi-16.

Damasio, A. R. (2004, April). *Emotions and Feelings*. Feelings and Emotions: The Amsterdam Symposium. https://doi.org/10.1017/CBO9780511806582.004

Damasio, A. R. (2000). The fabric of the mind: A neurobiological perspective. *Progress in Brain Research, 126*, 457–467. https://doi.org/10.1016/S0079-6123(00)26029-9

Damasio, A. R. (2018). *The Strange Order of Things: Life, Feeling, and the Making of Cultures: Vol. First edition.* Vintage.

Davidovitch, N., & Yavich, R. (2018). The Effect of Motivation and Self-Efficacy on Math Studies in the Israeli Ministry of Education's Program "Give Five." *International Journal of Higher Education*, *7*(6), 63–77.

Davydov, V. V. (2008). *Problems of Developmental Instruction: A Theoretical and Experimental Psychological Study*. Nova Science

Deci, E. L., & Ryan, R. M. (2000). The "What" and "Why" of Goal Pursuits: Human Needs and the Self-Determination of Behavior. *Psychological Inquiry, 11*(4), 227. https://doi-org.ezp.sub.su.se/10.1207/S15327965PLI1104_01

Desoete, A., Baten, E., Vercaemst, V., De Busschere, A., Baudonck, M., & Vanhaeke, J. (2019). Metacognition and motivation as predictors for mathematics performance of Belgian elementary school children. *ZDM*, *51*(4), 667–677. https://doi.org/10.1007/s11858-018-01020-w

Di Martino, P. (2019). Pupils' view of problems: The evolution from kindergarten to the end of primary school. *Educational Studies in Mathematics*, *100*(3), 291–307. https://doi.org/10.1007/s10649-018-9850-3

Dobie, T. E. (2019). A Sociocultural Examination of Utility Value in Mathematics: The Role of Interdependence in Middle School Students' Perceptions of Usefulness. In M. S. Hannula, G. C. Leder, F. Morselli, M. Vollstedt, & Q. Zhang (Eds.), *Affect and Mathematics Education: Fresh Perspectives on Motivation, Engagement, and Identity* (pp. 67–88). Springer International Publishing. https://doi.org/10.1007/978-3-030-13761-8_4

Dweck, C. S., & Leggett, E. L. (1988). A social-cognitive approach to motivation and personality. *Psychological Review, 95*(2), 256-273. doi:http://dx.doi.org.ezp.sub.su.se/10.1037/0033-295X.95.2.256

Edling, C. & Hedström, P. (2003). Kvantitativa metoder: grundläggande analysmetoder för samhälls- och beteendevetare. Lund: Studentlitteratur.

Eriksson, K., Björnstjerna, M., & Vartanova, I. (2020). The Relation Between Gender Egalitarian Values and Gender Differences in Academic Achievement. In *Frontiers in Psychology* (Vol. 11). Frontiers in Psychology. http://urn.kb.se/resolve?urn=urn:nbn:se:su:diva-180395

Evans, J. (2000). *Adults' Mathematical Thinking and Emotions: A Study of Numerate Practice*. Routledge.

Frid, S., Sumpter, L. & Nortvedt, G. (2020). Who is best in mathematics? In *Interim Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*. Inprasitha, M., Changsri, N. & Boonsena, N. (Eds). Khon Kaen, Thailand: PME.

Giota, J. (2013). Individualiserad undervisning i skolan – en forskningsöversikt. *Vetenskapsrådets rapportserie,* 3:2013. Stockholm.

Goldin, G. A., Epstein, Y. M., Schorr, R. Y., & Warner, L. B. (2011). Beliefs and engagement structures: Behind the affective dimension of mathematical learning. *ZDM: The International Journal on Mathematics Education*, *43*(4), 547. https://doi.org/10.1007/s11858-011-0348-z

Hannula, M. S. (2012). Exploring new dimensions of mathematics-related affect: Embodied and social theories. *Research in Mathematics Education*, *14*(2), 137–161.

Hannula, M. S. (2019). Young learners' mathematics-related affect: A commentary on concepts, methods, and developmental trends. *Educational Studies in Mathematics*, *100*(3), 309–316. https://doi.org/10.1007/s10649-018-9865-9

Hannula, M. S. (2011). *The structure and dynamics of affect in mathematical thinking and learning*. 34–60.

Hernandez-Martinez, P. (1), & Pampaka, M. (2). (2017). *"I did use to like maths...": Emotional Changes Toward Mathematics During Secondary School Education*. Elsevier Inc. https://doi.org/10.1016/B978-0-12-802218-4.00007-8

Heyd-Metzuyanim, E. (2019). Identity at the Crossroads, Collision or Conciliation? A Commentary. In M. S. Hannula, G. C. Leder, F. Morselli, M. Vollstedt, & Q. Zhang (Eds.), *Affect and Mathematics Education: Fresh Perspectives on Motivation, Engagement, and Identity* (pp. 419–428). Springer International Publishing. https://doi.org/10.1007/978-3-030-13761-8_19

Hultén, M. (2019). Striden om den goda skolan: Hur kunskapsfrågan enat, splittrat och förändrat svensk skola och skoldebatt. Lund: Nordic Academic Press.

Järvenoja, H., Järvelä, S., & Malmberg, J. (2017). Supporting groups' emotion and motivation regulation during collaborative learning. *Learning and Instruction*, 101090. https://doi.org/10.1016/j.learninstruc.2017.11.004

Karlsson, I. (2019). *Elever i matematiksvårigheter: Lärare och elever om låga prestationer i matematik* [Institutionen för utbildningsvetenskap, Lunds universitet]. https://portal.research.lu.se/portal/sv/publications/elever-i-matematiksvaarigheter(3e72c383-efa6-4a5d-aaa6-624d74d732bc).html

Kvale, S., & Brinkmann, S. (2014). *Den kvalitativa forskningsintervjun* (3rd ed.). Studentlitteratur.

Larsen, D. M. (2014). *Patterns of participation: A framework for understanding the role of the teacher for emerging classroom practices*.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge Univ. Press.

Leontyev, A. N. (n.d.). *Activity and Consciousness*. 196.

Lewis, G. (2013). Emotion and disaffection with school mathematics. *Research in Mathematics Education*, *15*(1), 70–86. https://doi.org/10.1080/14794802.2012.756636

Liljedahl, P. (2015). Emotions as an orienting experience. In K. Krainer & N. Vondrová (Eds.), *CERME 9 – Ninth Congress of the European Society for Research in Mathematics Education* (pp. 1223–1230). Charles University in Prague, Faculty of Education and ERME. https://hal.archives-ouvertes.fr/hal-01287349

Liu, Y., & Hou, S. (2018). Potential reciprocal relationship between motivation and achievement: A longitudinal study. *School Psychology International*, *39*(1), 38–55. https://doi.org/10.1177/0143034317710574

Lövheim, H. (2012). A new three-dimensional model for emotions and monoamine neurotransmitters. *Medical Hypotheses*, *78*(2), 341–348. https://doi.org/10.1016/j.mehy.2011.11.016

Madjar, N., Zalsman, G., Weizman, A., Lev-Ran, S., & Shoval, G. (2018). Predictors of developing mathematics anxiety among middle-school students: A 2-year prospective study. *International Journal of Psychology*, *53*(6), 426–432. https://doi.org/10.1002/ijop.12403

Marton, F., Hounsell, D. and Entwistle, N. (eds.) (1997). *The Experience of Learning* 2nd edition. Edinburgh: Scottish Academic Press.

Maturana, H. R., & Varela, F. J. (1992). *The tree of knowledge: The biological roots of human understanding*. Shambhala.

McGeown, S. P., & Warhurst, A. (2020). Sex differences in education: Exploring children's gender identity. *Educational Psychology*, *40*(1), 103–119. https://doi.org/10.1080/01443410.2019.1640349

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. *Handbook of Research on Mathematics Teaching and Learning*, *1*, 575–596.

Mujtaba, T., & Reiss, M. J. (2016). "I Fall Asleep in Class …But Physics Is Fascinating": The Use of Large-Scale Longitudinal Data to Explore the Educational Experiences of Aspiring Girls in Mathematics and Physics. *Canadian Journal of Science, Mathematics and Technology Education*, *16*(4), 313–330. https://doi.org/10.1080/14926156.2016.1235743

Nielsen, H. B. (2017). Chapter 15: Gender as Analytic, Political and Interdisciplinary Concept. *Concepts in Action*, 264–283. https://doi.org/10.1163/9789004314207_016

Nyman, M. (in press). Primary students' expressed emotions towards mathematics. In *Sustainable mathematics education in a digitalized world. Proceedings of MADIF 12*. SMDF.

Nyman, M. (in preparation). Secondary students' expressed emotion and motivation towards mathematics.

Nyman, M., & Sumpter, L. (2019). The issue of 'proudliness': Primary students' motivation towards mathematics. *LUMAT: International Journal on Math, Science and Technology Education, 7*(2), 80–96–80–96. https://doi.org/10.31129/LUMAT.7.2.331

Pekrun, R. (2006). The Control-Value Theory of Achievement Emotions: Assumptions, Corollaries, and Implications for Educational Research and Practice. *Educational Psychology Review*, *18*(4), 315–341. https://doi.org/10.1007/s10648-006-9029-9

Pekrun, R., Lichtenfeld, S., Marsh, H. W., Murayama, K., & Goetz, T. (2017). Achievement Emotions and Academic Performance: Longitudinal Models of Reciprocal Effects. *Child Development*, *88*(5), 1653–1670. https://doi.org/10.1111/cdev.12704

Radford, L. (2015). Of Love, Frustration, and Mathematics: A Cultural-Historical Approach to Emotions in Mathematics Teaching and Learning**.** B. Pepin, B. Roesken-Winter (eds.), *From beliefs to dynamic affect systems in mathematics education*, *Advances in Mathematics Education*. Springer International Publishing Switzerland. DOI 10.1007/978-3-319-06808-4_2

Ramirez, G. (2017). Motivated Forgetting in Early Mathematics: A Proof-of-Concept Study. *Frontiers in Psychology*, *8*. https://doi.org/10.3389/fpsyg.2017.02087

Ramirez, G., Shaw, S. T., & Maloney, E. A. (2018). Math Anxiety: Past Research, Promising Interventions, and a New Interpretation Framework. *Educational Psychologist*, *53*(3), 145–164. https://doi.org/10.1080/00461520.2018.1447384

Rodríguez, S., Regueiro, B., Piñeiro, I., Estévez, I., & Valle, A. (2020). Gender Differences in Mathematics Motivation: Differential Effects on Performance in Primary Education. *Frontiers in Psychology*, *10*. https://doi.org/10.3389/fpsyg.2019.03050

Rojo Robas, V., Villarroel Villamor, J. D., & Madariaga Orbea, J. M. (2018). The Affective Domain in Learning Mathematics According to Students' Gender. *Revista Latinoamericana de Investigación En Matemática Educativa*, *21*(2), 183–202. https://doi.org/10.12802/relime.18.2123

Ryan, R. M., & Deci, E. L. (2000). Intrinsic and Extrinsic Motivations: Classic Definitions and New Directions. *Contemporary Educational Psychology*, *25*(1), 54–67. https://doi.org/10.1006/ceps.1999.1020

Samuelsson, J. (2011). Development of self-regulated learning skills in mathematics in lower secondary school in Sweden. In *Nordisk matematikkdidaktikk: Vol. 16:3*. Nordisk matematikkdidaktikk. http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-74718

Schirmer, A. (2015). *Emotion*. SAGE Publications, Inc. http://sk.sagepub.com/books/emotion

Schoenfeld, A. H. (2020). Mathematical practices, in theory and practice. *ZDM: Mathematics Education, 1*. https://doi.org/10.1007/s11858-020-01162-w

Schukajlow, S., Rakoczy, K., & Pekrun, R. (2017). Emotions and motivation in mathematics education: Theoretical considerations and empirical contributions. *ZDM -SPRINGER-*, *3*, 307.

Schunk, D. H., Pintrich, P. R., & Meece, J. L. (2010). *Motivation in education: Theory, research, and applications* (3. ed., Internationl ed.). Pearson Education International.

Sekreter, G., & Serin, H. (2017). How Does Students' Sense of Self-Worth Influence Their Goal Orientation in Mathematics Achievement? *International Journal of Social Sciences & Educational Studies, 4(*3), 175–181. https://doi.org/10.23918/ijsses.v4i3p175

Sfard, A. (2005). What Could Be More Practical than Good Research? *Educational Studies in Mathematics*, *58*(3), 393.

Skolverket (2018, August) *Lista över socioekonomiskt index och den förväntade andelen obehöriga elever till gymnasieskolan.* https://www.skolverket.se/download/18.4fc05a3f164131a7418133b/1535448111495/Lista_socioekonomiskt_index-2019.pdf

Stoet, G., & Geary, D. C. (2018). The Gender-Equality Paradox in Science, Technology, Engineering, and Mathematics Education. *Psychological Science*, *29*(4), 581–593. https://doi.org/10.1177/0956797617741719

Suárez-Álvarez, J., Fernández-Alonso, R., & Muñiz, J. (2014). Self-concept, motivation, expectations, and socioeconomic level as predictors of academic performance in mathematics. *Learning & Individual Differences*, *30*, 118–123. https://doi.org/10.1016/j.lindif.2013.10.019

Sumpter, L. (2012) Upper secondary school students' gendered conceptions about mathematics. *Nordic studies in Mathematics Education, 17* (2), 27-47.

Sumpter, L. (2013). Themes and interplay of beliefs in mathematical reasoning. *International Journal of Science and Mathematics Education*, *11*(5), 1115–1135.

Sutter-Brandenberger, C. C., Hagenauer, G., & Hascher, T. (2018). Students' self-determined motivation and negative emotions in mathematics in lower secondary education – Investigating reciprocal relations. *Contemporary Educational Psychology*, *55*, 166–175. https://doi.org/10.1016/j.cedpsych.2018.10.002

Trezise, K., & Reeve, R. a. (2017). *The Impact of Anxiety and Working Memory on Algebraic Reasoning*. Elsevier Inc. https://doi.org/10.1016/B978-0-12-802218-4.00005-4

Tzohar-Rozen, M., & Kramarski, B. (2014). Metacognition, Motivation and Emotions: Contribution of Self-Regulated Learning to Solving Mathematical Problems. *Global Education Review*, *1*(4). https://ger.mercy.edu/index.php/ger/article/view/63

Vetenskapsrådet. (2017). *God forskningssed* [Good research practice]. Vetenskapsrådets rapportserie 1:2001.

Vinni-Laakso, J., Guo, J., Juuti, K., Loukomies, A., Lavonen, J., & Salmela-Aro, K. (2019). The Relations of Science Task Values, Self-Concept of Ability, and STEM Aspirations Among Finnish Students From First to Second Grade. *Frontiers in Psychology*, *10*. https://doi.org/10.3389/fpsyg.2019.01449

Wedege, T. (2011). Doing gender in mathematics education. In G. Brandell & A. Pettersson (Eds.), *Matematikundervisning: Vetenskapliga perspektiv* (pp. 92–114. Stockholm: Stockholm universitets förlag.

Yavuz, G. (2018). Mathematics Anxiety of Ninth Grade Students. *Journal of Education and Training Studies, 6*(5), 21–27.

Young, M. (2015). What is Learning and Why Does It Matter? *European Journal of Education*, *1*, 17.

Appendices

Appendix A: Consent form for respondents participating in Study I.

Appendix B: Consent form for respondents participating in Study II. Sub 15 years version.

Appendix C: Consent form for respondents participating in Study II. Over 15 years version.

Appendix D: Questionnaire used in Study II.

Stockholms universitet

Medgivande till deltagande i forskningsprojekt

Mitt namn är Martin Nyman och jag är doktorand vid Stockholms Universitet, Institutionen för Matematikämnets och Naturvetenskapsämnenas Didaktik (MND). I min studie kommer jag undersöka elevers uppfattningar och tankar om matematik och matematikundervisning. Jag kommer göra korta intervjuer med elever och analysera dem med fokus på deras motivation och attityder. Intervjuerna kommer att äga rum på ditt barns skola någon gång under skoldagen och de kommer spelas in (endast ljud, inte video) och därefter transkriberas, de intervjuade eleverna kommer anonymiseras i avhandlingen.
Jag ber härmed om tillstånd att genomföra en sådan kort intervju med ditt barn.

Jag utgår från Vetenskapsrådets forskningsetiska principer vid genomförandet av studien, materialet kommer efter att avhandlingen publicerats att arkiveras på ett sätt i enlighet med forskningsetiska principer angivna i Vetenskapsrådets *God Forskningssed*. Under arbetet är det endast jag själv och mina handledare som kommer ha tillgång till materialet.

Deltagandet i undersökningen är frivilligt och kan avbrytas när som helst utan att någon särskilt anledning måste uppges.

Med vänliga hälsningar

Martin Nyman
martin.nyman@mnd.su.se
072-529 47 22

Handledare vid Stockholms Universitet:

Lovisa Sumpter
lovia.sumpter@mnd.su.se

Paul Andrews
paul.andrews@mnd.su.se

Jag har tagit del av ovanstående information och godkänner att mitt barn deltar i undersökningen.

Barnets namn: _____ Datum _____

Målsmans underskrift: _____

Namnförtydligande: _____

Institutionen för matematikämnets och naturvetenskapsämnenas didaktik

Stockholms universitet
Institutionen för matematikämnets och naturvetenskapsämnenas didaktik
Stockholms universitet
106 91 Stockholm

Besöksadress:
Svante Arrhenius väg 20 A
114 18 Stockholm
www.mnd.su.se

Telefon: 08-16 20 00
Telefax: 08-1207 6570

E-post: registrator@mnd.su.se

Stockholms universitet

Samtycke till medverkan i forskningsprojekt

Bästa vårdnadshavare,

Ert (ditt) barns skola kommer under läsåret 2018/2019 att delta i ett forskningsprojekt kring uppfattningar, attityder, känslor och motivation i relation till matematik.
Eleverna har fått muntlig information om detta projekt under skoltid och har då också haft möjlighet att ställa frågor om projektet.

Syfte med forskningsprojektet
Syftet med forskningsprojektet är att med hjälp av enkäter och uppföljande intervjuer få en såväl kvantitativ som kvalitativ bild av högstadieelevers uppfattning av vad matematik är, vad syftet kan vara med att lära sig det och hur de upplever arbetet med matematik. Målet är att studien ska bidra till att bättre förstå anledningarna till att många elever tycks tappa intresse och motivation för matematik under sin skoltid.
Projektet drivs som en del av en doktorandutbildning vid Stockholms Universitet i samarbete med två disputerade handledare.

Datainsamling och datahantering
Ert barn kommer tillsammans med sina kamrater i klassen att genomföra en skriftlig enkät. Några av de som vill kommer också att intervjuas vid ett senare tillfälle. Intervjuerna kommer att dokumenteras med ljudinspelningar och de kommer att genomföras enskilt. Varje intervju tar ca 30 minuter. All datainsamling kommer genomföras i skolan, under lektionstid. Efteråt kommer intervjuerna att transkriberas och i samband med det kommer alla namn att fingeras. Barnens riktiga namn kommer bara finnas på ljudinspelningarna.

Alla svar på enkäterna samt såväl ljudinspelningar som transkriptioner kommer att förvaras på ett sådant sätt att inte obehöriga kan ta del av dem. Allt datamaterial kommer att hanteras enligt gällande forskningsetiska riktlinjer och datamaterialet kommer när projektet är avslutat att sparas enligt Arkivlagen och Riksarkivets allmänna föreskrifter. Tiden är vanligen tio år efter att projektet är avslutat.

Studiens resultat kommer att tillgängliggöras i form av till exempel tidskriftsartiklar, böcker och konferenspresentationer.

Mer information om de forskningsetiska principerna finns via:
http://www.codex.vr.se/manniska2.shtml.

Institutionen för matematikämnets och naturvetenskapsämnenas didaktik

Stockholms universitet
Institutionen för matematikämnets och naturvetenskapsämnenas didaktik
106 91 Stockholm

Besöksadress:
Svante Arrhenius väg 20 A
114 18 Stockholm
www.mnd.su.se

Telefon: 08-16 20 00
Telefax: 08-1207 6570

E-post: registrator@mnd.su.se

Stockholms universitet

Hantering av personuppgifter
Det som enligt den nya lagen (EU:s dataskyddsförordning 2016/679, GDPR) räknas som personuppgifter i denna studie är de ljudinspelningar där det går att identifiera de enskilda eleverna. Personuppgiftshanteringen sker med stöd av givet samtycke.

Ert barns deltagande i studien är frivilligt och ni som vårdnadshavare har rätt att

- när som helst under studien återkalla ett givet samtycke
- begära tillgång till ert barns personuppgifter
- få personuppgifterna rättade
- få personuppgifterna raderade
- få behandlingen av personuppgifterna begränsade
- inge klagomål till datainspektionen

Personuppgiftsansvarig är Stockholms universitet. Om ni önskar avbryta eller på annat sätt ändra givet samtycke går det bra att kontakta någon av de medverkande forskarna (se nedan). Det går också bra att kontakta universitetets dataskyddsombud, Benita Falenius, mailto:dso@su.se, tel. 08-16 41 91.

Vi hoppas att ni som vårdnadshavare, liksom vi, ser värdet av denna forskning och därmed samtycker till att ert barn deltar i projektet. På nästa sida finns en samtyckesblankett.

Martin Nyman, doktorand

Övriga medverkande är:
Handledare Lovisa Sumpter, Docent
Handledare Kerstin Larson, Fil Dr

Vänligen besvara denna samtyckesblankett och returnera den till undervisande matematiklärare **snarast**.

Stockholms universitet

Samtyckesblankett

Mitt barn har fått muntlig information och har haft möjlighet att ställa frågor. Vi (jag) har tagit del av den skriftliga information om studien och vi som vårdnadshavare får behålla den skriftliga informationen.

OBS! I och med de nya personuppgiftsreglerna (GDPR) måste samtliga vårdnadshavare samtycka till att minderårigt barn (under 15 år) deltar i ett forskningsprojekt.

☐ Vi samtycker till att vårt barn _____[namn] deltar i studien om uppfattningar, attityder, känslor och motivation i relation till matematik och till uppgifter om vårt barn behandlas på det sätt som beskrivs ovan.

☐ Vi samtycker INTE till att vårt barn deltar i studien.

Ort och datum	Underskrift vårdnadshavare 1

Ort och datum	Underskrift vårdnadshavare 2

Stockholms universitet

Samtycke till medverkan i forskningsprojekt

Bästa elev,

Din skola kommer under läsåret 2018/2019 att delta i ett forskningsprojekt kring uppfattningar, attityder, känslor och motivation i relation till matematik.

Syfte med forskningsprojektet
Syftet med forskningsprojektet är att med hjälp av enkäter och uppföljande intervjuer få en såväl kvantitativ som kvalitativ bild av högstadieelevers uppfattning av vad matematik är, vad syftet kan vara med att lära sig det och hur de upplever arbetet med matematik. Målet är att studien ska bidra till att bättre förstå anledningarna till att många elever tycks tappa intresse och motivation för matematik under sin skoltid.
Projektet drivs som en del av en doktorandutbildning vid Stockholms Universitet i samarbete med två disputerade handledare.

Datainsamling och datahantering
Du kommer tillsammans med sina kamrater i klassen att genomföra en skriftlig enkät. Några av de som vill kommer också att intervjuas vid ett senare tillfälle. Intervjuerna kommer att dokumenteras med ljudinspelningar och de kommer att genomföras enskilt. Varje intervju tar ca 30 minuter. All datainsamling kommer genomföras i skolan, under lektionstid. Efteråt kommer intervjuerna att transkriberas och i samband med det kommer alla namn att fingeras. Alla deltagares riktiga namn kommer bara finnas på ljudinspelningarna.

Alla svar på enkäterna samt såväl ljudinspelningar som transkriptioner kommer att förvaras på ett sådant sätt att inte obehöriga kan ta del av dem. Allt datamaterial kommer att hanteras enligt gällande forskningsetiska riktlinjer och datamaterialet kommer när projektet är avslutat att sparas enligt Arkivlagen och Riksarkivets allmänna föreskrifter. Tiden är vanligen tio år efter att projektet är avslutat.

Studiens resultat kommer att tillgängliggöras i form av till exempel tidskriftsartiklar, böcker och konferenspresentationer.

Mer information om de forskningsetiska principerna finns via:
http://www.codex.vr.se/manniska2.shtml.

Institutionen för matematikämnets och naturvetenskapsämnenas didaktik

Stockholms universitet
Institutionen för matematikämnets och naturvetenskapsämnenas didaktik
106 91 Stockholm

Besöksadress:
Svante Arrhenius väg 20 A
114 18 Stockholm
www.mnd.su.se

Telefon: 08-16 20 00
Telefax: 08-1207 6570

E-post: registrator@mnd.su.se

Stockholms universitet

Hantering av personuppgifter
Det som enligt den nya lagen (EU:s dataskyddsförordning 2016/679, GDPR) räknas som personuppgifter i denna studie är de ljudinspelningar där det går att identifiera de enskilda eleverna. Personuppgiftshanteringen sker med stöd av givet samtycke.

Ditt deltagande i studien är frivilligt och du har rätt att

- när som helst under studien återkalla ett givet samtycke
- begära tillgång till dina personuppgifter
- få personuppgifterna rättade
- få personuppgifterna raderade
- få behandlingen av personuppgifterna begränsade
- inge klagomål till datainspektionen

Personuppgiftsansvarig är Stockholms universitet. Om du önskar avbryta eller på annat sätt ändra givet samtycke går det bra att kontakta någon av de medverkande forskarna (se nedan). Det går också bra att kontakta universitetets dataskyddsombud, Benita Falenius, mailto:dso@su.se, tel. 08-16 41 91.

Vi hoppas att du precis som vi, ser värdet av denna forskning och därmed samtycker till att delta i projektet. På nästa sida finns en samtyckesblankett.

Martin Nyman, doktorand

Övriga medverkande är:
Handledare Lovisa Sumpter, Docent
Handledare Kerstin Larson, Fil Dr

Stockholms universitet

Samtyckesblankett

Jag har fått muntlig information och har haft möjlighet att ställa frågor. Jag har också tagit del av den skriftliga informationen om studien och får behålla den skriftliga informationen.

☐ Ja, jag samtycker till att delta i studien om uppfattningar, attityder, känslor och motivation i relation till matematik och till att uppgifterna om mig behandlas på det sätt som beskrivs ovan.

☐ Nej, jag samtycker INTE till att delta i denna studie.

_____
Ort och datum

_____
Underskrift

_____
Namnförtydligande

Enkät

Ringa in det som stämmer på dig

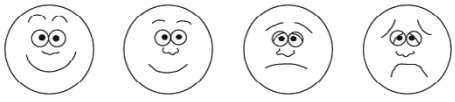
Jag går i årskurs 2 5 8 9

Jag är: flicka pojke

1 Så tycker jag om matte (ringa in det ansikte som passar bäst)

2 Så här känner jag inför en mattelektion (ringa in det ansikte som passar bäst)

3 Så här känner jag inför en svenskalektion (ringa in det ansikte som passar bäst)



4 Därför gör jag matte (svara med egna ord)

5 När jag gör matte känner jag så här (svara med egna ord)

6 Det här gör jag när jag gör matte (svara med egna ord)

7 Så här ser det ut när jag gör matte – rita en bild (använd baksidan på papperet)

I

The issue of 'proudliness': Primary students' motivation towards mathematics

Martin Nyman and Lovisa Sumpter

Department of Mathematics and Science Education, Stockholm University, Sweden

In this paper, we study year 2 and year 5 students' expressed motivations for doing mathematics. The responses were analysed using thematic analysis; first with a deductive approach using themes from previous research, and then an additional inductive analysis searching for new themes. The results show that the children express both intrinsic motivation (cognitive-oriented and emotional-oriented), as well as extrinsic motivation (including outward and compensation). Two new categories of cognitive intrinsic motivation were found—normative and personal. The results also indicated an interplay not only between the different categories but also within categories, signalling that expressed motivation is double-layered. Some implications are discussed.

Keywords
affect,
mathematics education,
motivation,
primary students

Correspondence
martin.nyman@mnd.su.se

DOI
https://doi.org/10.31129/LUMAT.7.2.331

1 Introduction

Motivation is a central component of learning, including mathematics learning (Gerholm, 2016; Schukajlow, Rakoczy, & Pekrun, 2017). Simply put, "to understand students' behaviour we need to know their motives" (Hannula, 2006, p. 165). A reduced description is that if learning is to take place, we need some incentive (Ryan & Deci, 2000a), and this incentive is pivotal if students are to direct their behaviour towards learning (Radford, 2015). Therefore, it is relevant to investigate individuals' expressed motivations if we want to understand why, for instance, some students are positively or negatively disposed towards mathematics, or understand the differences in performance in mathematical tests. One possible consequence of students' perceiving maths to be hard, boring and useless is students' choosing not to continue studying mathematics as soon as they are given a chance to opt out (e.g. Brown, Brown, & Bibby, 2008).

Even though there has been much research in psychology and general education generating several theories aiming to support explanations about motivation and learning, such as Bandura's (1977) theory about self-efficacy or Wigfield and Eccles' (2000) expectancy-value theory, the topic has not received much attention in mathematics education (Hannula, 2006; Schukajlow et al., 2017). Two relatively recent studies that focus on younger students' motivation for learning mathematics—year 2 (age 8) and year 5 (age 11)—both report that most students in year 2 express

mainly positive motivation towards mathematics whereas students in year 5 express significantly more negative motivation, such as stress (Blomqvist, Elamari & Sumpter, 2012; Dahlgren, Johansson & Sumpter, 2010). This contrasted with an earlier Swedish report that found interests and positive emotions peaked in grade 5 (Skolverket, 2003). Such results, potentially indicating a rather rapid change in expressed motivations and emotions, signal the need to know more about young students' motivations and other affective constructs. Therefore, the aim of this paper is to explore a qualitatively nuanced understanding of expressed motivation and potential relationships between different affect-related constructs. The following research questions are posed: (1) "What different types of motivation do primary school students express?" and (2) "How are these different types of motivation interrelated in the students' responses?"

2 Theoretical background

As stated earlier, motivation has been of academic interest for decades, and we can only give a limited background. Motivation can be understood as 'the engine that keeps us going', the drive to accomplish things that in one way or another are of some importance to us. More specifically, one definition is: when a person "is energized or activated toward an end" (Ryan & Deci, 2000b, p. 54) and inversely, when you feel no drive or inspiration to act you are considered unmotivated. Another definition:

> "Motivation is the process whereby goal-directed activity is instigated and sustained." (Schunk, Pintrich, & Meece, 2010, p. 4).

Both definitions relate motivation to a goal or an end which implies it to be understood as conscious, although not necessarily positive, since a goal can be either positive—striving for attainment, or negative—seeking to avoid. The word 'activity', also used in both definitions, connects the concept of motivation to a starting point, meaning that motivation needs to be triggered somewhere and somehow (c.f. Ryan & Deci, 2000b; Schunk et al., 2010). One explanation for the origin of these triggers and the energy of driving forces is the concept of need (Hannula, 2006; Ryan & Deci, 2000a), and more specifically the need for relatedness, autonomy and competence as inherent driving forces for human motivation.

In Deci and Ryan's (1985) theory of self-determination the notions of extrinsic and intrinsic motivation are considered as parts of the same motivational spectrum that

range from amotivated, where a person has neither positive nor negative motivation, to extrinsically motivated, to intrinsically motivated. What distinguishes between these levels are the levels and origin of self-regulation, a construct primarily linked to autonomy, but also to competence in the form of self-efficacy. When we see people doing something for the pure satisfaction of doing it, because it is fun or challenging and not because it renders you any type of external consequence, this person is considered intrinsically motivated, as opposed to an extrinsically motivated person being active in order to attain a certain outcome (Ryan & Deci, 2000a). However, in the way most schools are organised, factors such as grades and exams are linked to extrinsic motivation. This means that teachers also need to regard extrinsic motivation as an influence on learning. It has been suggested that intrinsic and extrinsic motivation should not be perceived as a bipolar construct, at least not in classroom situations (Harter, 1981), and research has shown that elementary school children express both intrinsic and extrinsic motivation when asked about their motives for studying (Lepper, Corpus & Iyengar, 2005). In relation to achievement, Hattie (2009) establishes a link between motivation and achievement but without potential direction differentiation. Further work on the relation between motivation and achievement is done by Garon-Carrier et al. (2016). Their results indicate that there is no correlation between motivation and achievement, but that the correlation exists in the opposite direction—from achievement to motivation—and they discuss both temporal links as well as linkage to academic self-concept in relation to this result (Skaalvik, 1994). Hence the importance of understanding the various levels of both intrinsic and extrinsic motivation (Ryan & Deci, 2000a) as well as the potential interplay between the two alongside other factors (Hannula 2006, 2012; Prat-Sala & Redford, 2010).

Both intrinsic and extrinsic motivation can be further divided into subthemes (Amabile, Tighe, Hill & Hennessy, 1994) with the two main themes falling into two subscales each: extrinsic motivation is split between Compensation (e.g. personal gain such as grades) and Outward (e.g. personal appearance such as status). Further, Intrinsic motivation is divided between Challenge (e.g. trying to solve problems) and Enjoyment (e.g. positive feelings). However, when applying these subthemes to the study of upper secondary school students' indicated beliefs, Sumpter (2013) saw the need for an expansion of the intrinsic motivation subthemes. The subthemes were called Cognitive and Emotional respectively, allowing for more cognitive-linked

motivation such as wanting to learn and an expansion of the emotional spectrum to include the negative.

Also, studies focusing on students' motivation point towards constructs being intertwined or in some sort of interplay (e.g. Gerholm, 2016; Jansen, 2006). One example is the longitudinal case study of Rita, reported by Hannula (2002, 2006). Rita's comments concern emotions, beliefs and motivation in such an intertwined way that it seems impossible to separate them. Other research concludes that the constructs in themselves can be internally intertwined with their sub-constructs and hence difficult to separate. The case of Sam, an upper secondary school student reported by Sumpter (2013), illustrates how negative cognitive intrinsic motivation appears to be compensated by extrinsic motivation. Another study, focusing on high achieving students in upper secondary school, stresses that the feeling of joy that comes from being socially accepted has a pivotal motivational aspect (Gerholm, 2016). But the very opposite can also be true: students can actively avoid participating in social classroom activities for fear of producing a socially stigmatising image to peers or appear to the teacher to be a poor student (Jansen, 2006). This behaviour was linked to acting on negative compensational motivation, meaning that emotions and beliefs linked to motivation could be both negative and positive. Studies like these can illustrate how social factors are at play in school-related situations and activities, both perceived (Sumpter & Sternevik, 2013) and expressed in task-solving sessions (Sumpter, 2013). According to Radford (2015), understanding the connection between the individual and socio-cultural realm is of value:

> "/.../ the affective domain in general and motives and motivation in particular are not only subjective but also sociocultural phenomena. They are subjective and sociocultural in the sense that on the one hand, motives are the motives of a concrete and unique person but, on the other hand, they relate to a sociocultural and historical world that transcends the individual. In its transcendence, the sociocultural historical world indirectly – albeit in a decisive manner – shapes and organises the individual's motives and emotions. "(Radford, 2015, p. 26).

In this way, motivation can be considered an individual-based construct as well as something that is shaped and organised in interplay with the cultural and social environment where the individual exists and acts.

2.1 Methods

The present study seeks to build on the results from Blomqvist et al. (2012) and Dahlgren et al. (2010) where students in years 2 and 5 answered a seven-item questionnaire (for a complete list of the questions see end notes) and the data was analysed using mainly quantitative methods. This meant that the results were limited: both studies only reported generally using a positive/negative scale while nuances or other aspects of motivation were not explored and no possible explanation for the reported differences was offered. In this study we used Blomqvist et al.'s questions for semi-structured interviews in search of a qualitatively nuanced understanding of the motivational construct. Thus, we collected students' descriptions of their mathematics education experiences and of mathematics in general. The data consist of transcripts of interviews containing the respondents' explicit utterances. In addition, their implicit communication, including exclamations, sighs and pauses have been taken into account since this taps into a person's possibly subconscious driving forces (Bryman, 2016; Kvale & Brinkmann, 2014). As this study focusses on motivation, responses in relation to Blomqvist et al.'s question number four, "Why do you do maths?", will be discussed here. Our decision to use students from year 2 and year 5 enabled comparison between our results and those of Blomqvist et al. (2012) and Dahlgren et al. (2010).

2.2 Participants

Three schools were sampled for this study in a way that captured the distribution between city and suburban as well as the socio-economic settings of the urban area where this data was collected. At each school, teachers teaching the intended grades were asked to participate, their pupils were informed, and consent was obtained from parents. Shortly before each interview, the teachers were asked to pick out, from the group of volunteering pupils, individuals that they considered to be neither particularly good nor particularly poor at mathematics so we could get a large and diverse group that lay between the two extremes. The teachers were also asked to consider whether pupils would react well in the interview situation, to avoid some of the common difficulties associated with interviewing children (Hritz, Royer, Helm, Burd, Ojeda & Ceci, 2015). Prior to the present study a pilot study was conducted with six students from year 2, four students from year 4 and five students from year 6. The pilot study indicated that the questions worked on a general level but follow up

questions were needed especially when the respondents started talking about other subjects such as Star Wars or computer games.

All the interviews were conducted by the first author of this article over a period of four weeks in the participating schools during lesson time and in a separate room close to the classroom. In total 19 students participated—ten girls (four in year 2 and six in year 5) and nine boys (six in year 2 and three in year 5) were interviewed. Each interview lasted for about 25 minutes, including time for a drawing task (item number seven), and was audio-recorded. This limited sample size does not allow for any general conclusions, but it will be sufficient to see nuances in how students describe their experience of mathematics and their emotional and motivational relation to it (e.f. Guest, Bunce, & Johnson, 2006).

The ethical considerations stipulated by the Swedish Research Council through Codex (Vetenskapsrådet, 2017) were followed, meaning that all respondents had written parental consent and were informed that they were participating voluntarily and at any moment could stop the interview without having to provide any explanation. The Council also stipulates how data is managed and reported, and therefore all names have been changed.

2.3 Interviews

Semi-structured interviews allow follow up questions to be posed, providing the respondent with opportunities to provide richer detail (Bryman, 2016). The follow up questions were mainly of the "please expand" type, for instance "could you explain a bit further?" or "can you give an example?" but questioning also facilitated following a respondent's line of thought. This technique also allows the intended meaning of questions to be reciprocally verified; either the respondent asks the interviewer back, "[H]mm, what do you mean…?" or the interviewer rephrases the question when the response reveals a misunderstanding of the question. In addition to revealing nuances the follow up/please expand questions provide the interviewer with a method for triangulation through a set of responses around one topic, thus a means of countering common methodological issues associated with interviewing children, e.g. neutrality issues, interviewees feeling intimidated, etc. (Hritz et al., 2015; Talmy, 2011).

2.4 Methods of analysis

The interviews were transcribed verbatim, including non-verbal communication such as exclamations or stresses of words, as well as sighs and extended pauses. Instances where the respondent highlighted and stressed a word were marked with italics. Non-verbal instances of a more temporal nature such as pausing were marked with (...) when the pause was short, approximately less than two seconds, and with rectangular brackets ([thinks]) if the pause was longer. For readability the transcripts followed standard spelling conventions, for example, an utterance like: "ifju-no-wadda-meen" was transcribed as: "if you know what I mean". In this paper, when sections are omitted to shorten excerpts these places are marked with (/.../).

In the complete transcripts, responses to a selected question were marked and the analysis was subsequently carried out in three steps, the first two deductively and the third inductively (e.g Braun & Clarke, 2006). A first categorisation was made by connecting responses to either of the two main themes, extrinsic or intrinsic motivation (e.g. Ryan & Deci, 2000a). Coding for extrinsic motivation focussed on answers connected to the outside world, like: "you[everybody] have to know", or more explicitly connected with reward or punishment: "if I get a job as a cashier when I grow up" or "[if I can't do it] it will be embarrassing", respectively. The second theme was intrinsic motivation—interest, enjoyment and satisfaction for instance, where one example of a response could have been: "it feels good when I do it".

The second, deductive analytical phase used a four-part mapping scheme: first, data connected to the Extrinsic motivation category were mapped against the sub-themes Outward and Compensatory, here understood as social-gain values (e.g. "I'm concerned about what other people think of me.") and personal-gain values (e.g. "I'm motivated by what I can earn.") respectively, as described in Amabile et al. (1994). Second, data connected to the Intrinsic motivation category were mapped using the same subscales as described by Sumpter (2013) as linked to intrinsic motivation: Cognitive and Emotional. The subtheme Cognitive frames issues of knowledge and personal development, and Emotional contains statements like "I think it's fun doing it". In the third and last step of the analysis we followed the inductive thematic analysis as described by Braun and Clarke (2006), searching the total wealth of the data for recurring themes that captured "something important about the data /.../ and represent[ed] some level of *patterned* response ..." (Braun & Clarke, 2006, pp 82, italics original). One example was various references to "the future". However, further

analyses of the data reveal variations, like explicit mentions of "a job" or making "calculations", that contribute to form a larger and more nuanced pattern. This phase of the analysis is an iteration where potentially enhanced variation is balanced by a condensation of themes, looking for the point where the themes are still coherent and descriptive, but not overlapping. In the cases where a respondent's utterance contained more than one theme, the utterance was split.

The analysis was made primarily by the first author, and any unclear responses were analysed by the two authors separately before being discussed to increase the reliability of the analysis.

4 Results

First, as an overview and a guide for the reader, we present in the form of a table (Table 1), the different themes that resulted from the analysis, including the different subscales which informed the deductive analysis, and the subthemes which came out of the inductive analysis. Below, the contents of this table are discussed.

Table 1. Explicit motivation expressed by students in years 2 and 5. Outward to be interpreted as social-gain values and Compensation as personal-gain values. (Total number of instances within brackets).

Main themes	Subscales	Subthemes
Extrinsic Motivation	Outward (27)	Important for the future (12)
		To be able to calculate things (8)
		To get/manage a job (6)
		To produce an 'answer' (explicit) (1)
	Compensation (5)	Not to make calculation errors (3)
		You succeed if you make an effort (2)
Intrinsic motivation	Cognitive (15)	To learn (13)
		Want to try new things (1)
		Maths makes you better (1)
	Emotional (9)	Fun/I like it (7)
		Exciting (1)
		No-stress (double negative) (1)

Looking at Table 1, we see that the most common response (n=27) was regarding extrinsic motivation with the Outward subscale. These subthemes, including similarities and differences, will be discussed in three themes that reflect the result of the study. The aim of this paper is not to study differences between the two age groups.

However, because we believe that the respondents' age is an important part of the context, we have chosen to include this in the excerpts below.

4.1 Future needs—important but often vague

Most responses were about a specific work task or a profession in the future and were analysed under the Extrinsic motivation Outward scale. Louise uses the example of working at the till in a supermarket/shop:

> "I should learn so if I work at the till when I'm grown up for instance and there is a fruit that cost 20 crowns, and the person gives perhaps 40 crowns, then I should be able to calculate how much he/she should get in return." [Louise, Y2]

Here, it is about a specific situation (being able to calculate change at the till) more than it is about the profession 'shop assistant'. Anton, also year 2, talks about both the situation and the profession; here civil engineer:

> "To me it's like that you should learn for the future, if you will for instance have a job as ... an engineer or something because then you need to know a lot of maths; you need lengths and all. Shapes and figures."
> **Interviewer**: "...mm, and what do you need to use it for?"
> **Anton**: "To construct blueprints. And it's important to know mathematics." [Anton, Y2]

Anton not only states the profession but also gives examples of what types of mathematics (e.g. 'lengths') and why you need it (constructing blueprints). Often the responses are related to professions connected to being able to calculate things, for instance Melker (Y2) thinking about becoming a doctor: he "must know all the calculations" and it "has to be easy when you are grown up." The ability to calculate is also mentioned without links to any profession:

> "To be able to learn and so on. Because for example if it's your birthday and you made a cake and have invited seven people you must be able to split the cake in eight pieces and then you need the maths where you learnt to know how to split the cake." [Frida, Y5]

Minna has a very vague idea of any links at all, but nonetheless has a strong sense of necessity:

> "Because you have to learn. When you're grown up and you must pay, and if you work in a food store then you don't know how much money you should give

> back or how little. So, you have to know, anyway, to also know … you have to have maths." [Minna, Y2]

The situations described by Frida and Minna are more about everyday situations, but what these four examples (including Louise and Anton) have in common is the conviction that mathematics is very important for managing situations connected to adult life and responsibility. The statements above serve as examples of the Compensation category. A further analysis shows that some statements contain indications of a social rather than a personal reference, here illustrated by the following response:

> **Interviewer**: "Why do you do maths?"
> **Samira**: "What? Why I do maths?"
> **Interviewer**: "Yes, why do you do maths?"
> **Samira**: "At school?"
> **Interviewer**: "Yes, we can start there, why do you do maths at school?"
> **Samira**: "To learn. And you're going to have knowledge, it's good if you have it later in the future if you're going to buy stuff it's good to be able."
> **Interviewer**: "Ok. And why is it so good to be able?"
> **Samira**: "Because otherwise you can't know. For example, if I have a one-hundred bill and take more stuff than I can buy it'll be embarrassing at the till." [Samira, Y5]

In Samira's response, two matters are indicated. The first is the importance of knowing some mathematics in order to avoid embarrassing situations. The second is that due to this embarrassment, there is a risk that others might view you as a less knowledgeable person. In her response, there are traces of intrinsic motivation, negative emotions connected to avoidance, and outward extrinsic motivation.

4.2 "To learn" — of normative or personal origin

One common reason that pupils give for why they do mathematics, with slight variations, is 'to learn'. It could be an indication of extrinsic motivation such as you need to learn to be able to calculate, such as Frida's response above with an example of when and why, or it could be given without any further explanation. But it could also be about the individual's own wish, thus indicating a more intrinsic motivation. Comparing Pakisa's with Christopher's statements illustrates a delicate but important difference in their responses:

> **Pakisa**: "To learn how you calculate … and … I don't really know … you know, you learn maths and you need it in the future sometimes.
> **Interviewer**: "Why do you need it in the future?"
> **Pakisa**: [Thinks] "I don't know … "[Pakisa, Y5].

> **Christopher**: "I want to learn things. Try new things … and stuff." [Christopher, Y2]

Both statements reveal a connection to a cognitive dimension, but at the same time a subtle difference regarding intrinsic motivation; Pakisa's response is neither typically extrinsic, nor does it show the same intrinsic qualities as Christopher's does. In order to capture these slight differences in their expressed motivation, we chose to divide the subtheme 'To Learn' that is sorted under Intrinsic motivation and the subscale Cognitive (i.e. Intrinsic Motivation-Cognitive-To-Learn, shortened: ICL), into two: ICL-Personal and ICL-Normative. ICL-Personal is then defined as Intrinsic motivation with linkage to personal desires or issues of self-fulfillment whereas ICL-Normative is considered as Intrinsic motivation linked to convictions of right and wrong. In the above examples, Pakisa's statement is considered ICL-Normative based on "you need it in the future sometimes", implying that her driving force is based on complying with a norm, whereas Christopher's is considered ICL-Personal due to his "wanting" to learn. Here, the ICL-Normative group is dominant among the respondents. Some students offered both arguments when asked to differentiate between personal and normative and some stuck to one (either ICL-Personal or ICL-Normative), regardless of the way the questions were posed or whether the follow-up question took a slightly different track.

4.3 Interplay

Another result is the interplay of factors that can be interpreted as Intrinsic motivation with a mix of Cognitive and Emotional origin, here illustrated by Matteus and Casper:

> **Interviewer**: "Can you explain why you do math?"
> **Matteus**: "I think it … is because it's fun to calculate. You can take your time and calculate, it's no rush. You have the time."
> **Interviewer**: "Is that the *reason why* you do maths?"
> **Matteus**: [nods]
> **Interviewer**: "… because it's fun …?"
> **Matteus**: "Yes, it's also exciting."
> **Interviewer**: "How is it exciting?"
> **Matteus**: "… umm … if there's a task that is rather difficult, you don't think you're going to make it, but when you try you *do* make it." [Matteus, Y5]

In the above example Matteus' feeling of contentment is not explicitly expressed, but we sense it implicitly in his emphasis on the word '*do*'. The interplay is confirmed by Casper, from grade 2, who gives a similar but more explicit description:

> **Interviewer**: "Why do you do maths?"
> **Casper**: "Why I do maths?"
> **Interviewer**: "Mm?"
> **Casper**: "Because I like it very much ... and it just feels good when I do maths."
> **Interviewer**: "Aha, does it?"
> **Casper**: "Mm ..."
> **Interviewer**: "Can you try to describe that feeling of 'good'?"
> **Casper**: "It feels warming. Proudliness [Swedish: 'stoltlighet']. And it feels like you're going to make it." [Casper, Y2]

Here the motivation for doing maths is the "warming" feeling of pride when you manage to complete a (difficult) task. This is indicating an interplay between motivation and emotion.

Both Casper and Matteus express a similar motivational combination: one part being a positive and Emotion-driven motivation linked to Cognitive aspects: "it just feels good" and "it's fun" respectively, and the other part expressed as Extrinsic motivation—that there are tasks that require calculation and you have time to do this. But the last quote, from Matteus: "because it's fun to calculate, you can take your time and calculate, it's no rush. You have the time" at the same time reflects a third and temporal type of motivation: that if you do not have the time and you have to rush when calculating the tasks, then different emotions, most likely negative ones, also come into play. This positively expressed type of interplay consists of missing a negative emotion, borrowing from the arithmetic rule of thumb: "two minuses make a plus", we termed this category 'double negative'.

Figure 1 summarises the relationships between the different constructs in Table 1.

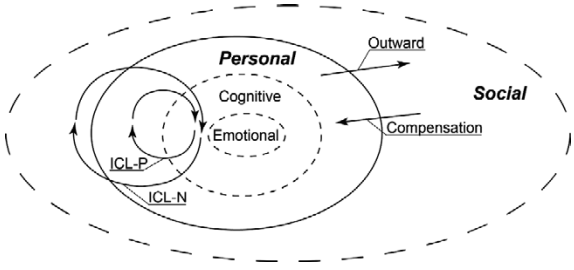


Figure 1. the relationships between the different constructs.

Figure 1 illustrates how the three Emotional and the three Cognitive components are located within the personal where the Emotional dimensions could be seen as part of a core concept. The two new constructs ICL-Personal and ICL-Normative both connect the cognitive components with the other components, here seen as an interplay. The two constructs related to extrinsic motivation, Outward and Compensation, can also be understood as an interplay between the personal and the social levels and with the two constructs working in opposite direction as each other. The Outward theme is the persons' motive to make social gains, and the Compensation theme is the person's motive to make some personal gain from the social or outside world.

5 Discussion

The aim of this paper was to study younger students' expressed motivation for doing mathematics. Looking at the main motivations that these students expressed, the responses are about future needs associated with the responsibilities of adulthood, some explicitly in connection with a profession or job, some in relation to shopping and some in very general terms, expressed as "I need it". While all respondents communicated the innate importance of mathematics, they sometimes struggled to give examples. This is somewhat different to previous studies where the most common response was mainly intrinsic motivation (Blomquist et al., 2012; Dahlgren Johansson & Sumpter, 2010). One explanation for this difference could be attributed to different research methods. In this study, the interview format and the possibility that the posing of follow-up questions allowed the respondents to offer several answers which deepened their explanations, thereby giving multiple motives for action. In some cases, this led to a seeming ambiguity in the responses, as illustrated with Samira's explanation about shopping. This ambiguity indicated that the categories Outward and Compensation could either be expanded in their definitions to include social situations or be expanded with a third category that focusses on social situations (c.f. Amabile et al., 1994). Here, based on our limited results, we cannot make any definitive conclusions. It would require additional investigations in order to determine whether it is different from the previous two categories or a refinement of the definitions. The result—that students in this study assign personal as well as social components to mathematical skills—is interesting given it has been noted in previous studies, both with prospective teachers (Sumpter & Sternevik, 2013) and upper-secondary school students (Sumpter, 2013), and it could be seen as a

confirmation of Radford's (2015) conclusion about the importance of social and cultural environments in motivation to study. Also, the findings of two new subthemes, ICL-Personal and ICL-Normative, means that a further investigation of sub-constructs could lead to a better understanding of what motivation is and how it can be expressed.

When looking at the Intrinsic category, we see that the subthemes Cognitive and Emotional were present just as in Sumpter (2013). In the present paper, the results also reveal nuances within each category. One example is the difference between joy and excitement. Both could be considered emotionally positive, but there is a difference between enjoying a mathematical problem and being excited and challenged by problem solving. Some students describe finding the working situation enjoyable because it is nicely framed, which is different to being excited by the prospect of winning a multiplication game or challenged by solving a tricky problem. Another, and perhaps complex, example from the Intrinsic category is the subtheme No-stress. This can be considered a double negative; the first negative being the absence of stress, and then, defining stress itself as a negative emotion, hence double negative. It should be emphasised that this No-stress theme is something other than the other two: the first two are clearly positive, albeit for different reasons, but this third theme is an interpretation of positive because it lacks negative loading. Given that the No-stress theme is linked to a comparison of previous mathematics schooling, it could be interesting to study older students' expressed motivation since they would have more experience to draw upon.

The combined conclusions around intrinsic and extrinsic motivational factors, as discussed above, is that even though they are very helpful in providing a tool for separating the inner from the outer sources of motivation, these concepts do not seem as separable and linear as those presented by Ryan and Deci (2000a; 2000b). Rather, the results from this study support the idea that the intrinsic/extrinsic constructs are intertwined and "messy", both in relation to each other and also to other affective constructs like emotion (c.f. Hannula, 2006, 2012; Radford, 2015).

When analysing the relationship between different types of motivation in the students' responses we found instances of interplay, such as being proud (i.e. Emotional) when solving a very difficult problem (i.e. Cognitive) or feeling embarrassed (i.e. Emotional) when being in a social situation (here Outward Extrinsic motivation). Our results support research that has looked at the interplay between intrinsic and extrinsic motivation (e.g. Lepper et al., 2005; Prat-Sala & Redford,

2010). But the link between other affective constructs such as emotions are also present, and just as Hannula (2012) concluded, it can be hard to separate them. Here it is illustrated by Casper's own invented word 'proudliness' ('Stoltlighet'). His strong feeling of pride when completing a challenging task appears to be so important to him that he invents a word appropriate for the occasion. This could be seen as a confirmation of Emotional being a subtheme under Intrinsic motivation, but also that emotions are an integral part of motivation, making interplay between affective constructs a probable by-product (c.f. Hannula 2012; Prat-Sala & Redford, 2010; Sumpter, 2013). One conclusion is that students' expressed motivation seldom appears to be one-dimensional, and one possible implication could then be that mathematics teaching cannot approach students' motivation in a one-dimensional way. Based on the qualitative differences themes within the Extrinsic category, we then suggest that researchers and teachers might need to reevaluate the role of motivation in mathematics education—that extrinsic motivation is also important, and constantly striving for students' positive intrinsic motivation could result in missing out on other motives and needs that are important.

Note

The questions in the questionnaire from Blomqvist et al. (2012) were:

1. What do you think about maths?
2. How do you feel before a maths lesson?
3. How do you feel before a Swedish lesson?
4. Why do you do maths?
5. How do you feel when you do maths?
6. What do you do when you do maths?
7. Please draw a picture of yourself when doing maths.

References

Amabile, T. M., Tighe, E. M., Hill, K. G., & Hennessey, B. A. (1994). The work preference inventory: Assessing intrinsic and extrinsic motivational orientations. *Journal of Personality & Social Psychology, 66*(5), 950–967. http://dx.doi.org.ezp.sub.su.se/10.1037/0022-3514.66.5.950

Bandura, A. (1977). Self-efficacy: Toward a unifying theory of behavioural change. *Psychological Review, 84*(2), 191–215. DOI: http://dx.doi.org.ezp.sub.su.se/10.1037/0033-295X.84.2.191

Barton, K. C. (2015) Elicitation Techniques: Getting people to talk about ideas they don't usually talk about, *Theory & Research in Social Education, 43:2*, 179–205, DOI: 10.1080/00933104.2015.1034392

Blomqvist, A., Elamari, U., & Sumpter, L. (2012). Grade 2 and Grade 5 students' conceptions about mathematics and mathematics education. In G.H. Gunnarsdottir, F. Hreinsdottir, G.

Palsdottir, M. Hannula, M. Hannula-Sormunen, E. Jablonka, U. Jankvist, A. Ryve, P. Valero & K. Wæge (Eds.), *Proceedings of NORMA 11: The sixth Nordic conference on mathematics education (pp. 187 – 196)*. Reykjavik

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Brown, M., Brown, P., & Bibby, T. (2008). "I would rather die": reasons given by 16-year-olds for not continuing their study of mathematics. *Research in Mathematics Education, 10*(1), 3–18. DOI: 10.1080/14794800801915814

Bryman, A. (2016) *Social research methods*, (Fifth edition). Oxford: Oxford University Press

Dahlgren Johansson, A. & Sumpter, L. (2010). Children's conceptions about mathematics and mathematics education. In K. Kislenko (Ed.) *Current state of research on mathematical beliefs XVI* (*pp.77 – 88*). Tallinn: OÜ Vali Press.

Deci, E. L., & Ryan, R. M. (1985). *Intrinsic motivation and self-determination in human behaviour*. New York: Plenum.

Garon-Carrier, G., Boivin, M., Guay, F., Kovas, Y., Dionne, G., Lemelin, J-P., Séguin, J. R., Vitaro, F. & Tremblay, R. E. (2016). Intrinsic Motivation and Achievement in Mathematics in Elementary School: A Longitudinal Investigation of Their Association

Gerholm, V. (2016). *Matematiskt begåvade ungdomars motivation och erfarenheter av utvecklande verksamheter. (Mathematically talented teenagers' motivation and experience of developmental activities.)* Licentiatuppsats. Rapporter i matematikämnets och naturvetenskapsämnenas didaktik, 8. Stockholm: Stockholms Universitet

Guest, G., Bunce, A., & Johnson, L. (2006). How many interviews are enough? An experiment with data saturation and variability. Great Britain: Sage Publications INC. https://doi-org.ezp.sub.su.se/10.1177/1525822X05279903v

Hannula, M. S. (2002). Attitudes towards mathematics: Emotions, expectations and values. *Educational Studies in Mathematics 49*: 25–46. Kluwer Academic Publishers. Netherlands

Hannula, M. S. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics 63*, 165–178. DOI: 10.1007/s10649-005-9019-8

Hannula, M. S. (2012). Exploring new dimensions of mathematics-related affect: Embodied and social theories. *Research in Mathematics Education, 14*(2), 137–161. https://doi-org.ezp.sub.su.se/10.1080/14794802.2012.694281

Harter, S. (1981). A new self-report scale of intrinsic versus extrinsic orientation in the classroom: Motivational and informational components. *Developmental Psychology, 17,* 300–312. DOI: 10.1037/0012-1649.17.3.300

Hattie, J. (2010). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. New York: Routledge.

Hritz, A. C., Royer, C. E., Helm, R. K., Burd, K. A., Ojeda, K., & Ceci, S. J. (2015). *Children's suggestibility research: Things to know before interviewing a child*. Anuario de Psicología Jurídica, 25, 3 – 12. https://doi-org.ezp.sub.su.se/10.1016/j.apj.2014.09.002

Jansen, A. (2006). Seventh Graders' Motivations for Participating in Two Discussion-Oriented Mathematics Classrooms *The Elementary School Journal, 106*(5), 409–428. The University of Chicago Press. http://dx.doi.org.ezp.sub.su.se/10.1086/505438

Kvale, S. & Brinkmann, S. (2014) *Den kvalitativa forskningsintervjun*. (3. [rev] ed.) Lund: Studentlitteratur

Lepper, M. R., Corpus, J. H., & Iyengar, S. S. (2005). Intrinsic and extrinsic orientations in the classroom: Age differences and academic correlates. *Journal of Educational Psychology, 97,* 184–196.

Prat-Sala, M. & Redford, P. (2010). The interplay between motivation, self-efficacy, and approaches to studying. *British Journal of Educational Psychology, 80*, 283–305. http://dx.doi.org.ezp.sub.su.se/10.1348/000709909X480563

Radford, L. (2015). Of love, frustration, and mathematics: A cultural-historical approach to emotions an mathematics teaching and learning. In B. Pepin, B. Roesken-Winter (eds.). *From beliefs to dynamic affect systems in mathematics education, Advances in mathematics education (pp. 25–49).* Switzerland, Springer international Publishing. DOI: 10.1007/978-3-319-06808-4_2

Ryan, R. M., & Deci, E. L. (2000a). Intrinsic and Extrinsic Motivations: Classic Definitions and New Directions. *Contemporary Educational Psychology, 25*, 54–67. DOI: 10.1006/ceps.1999.1020

Ryan, R. M., & Deci, E. L. (2000b). Self-determination theory and the facilitation of intrinsic motivation, social development, and well-being. *American Psychologist, 55*(1), 68.

Schukajlow, S., Rakoczy, K., & Pekrun, R. (2017). Emotions and motivation in mathematics education: Theoretical considerations and empirical contributions. Germany: Springer Science + Business Media.

Schunk, D. H., Pintrich, P. R., & Meece, J. L. (2010). *Motivation in education: Theory, research, and applications (3. ed., International ed.).* Upper Saddle River, N.J.: Pearson Education International.

Skaalvik, E. M. (1994). Attribution of perceived achievement in school in general and in math and verbal areas: Relations with academic self-concept and self-esteem. *British Journal of Educational Psychology,* 64, s. 133–143.

Skolverket (2003). *Lusten att lära [Joy to learn]* Stockholm: Liber

Sumpter, L. (2013). Themes and interplay of beliefs in mathematical reasoning. *International Journal of Science and Mathematics Education, 11*(5), 1115–1135. DOI: 10.1007/s10763-012-9392-6

Sumpter, L. & Sternevik, E. (2013). Prospective teachers' conceptions of what characterize a gifted student in mathematics. In M.S. Hannula, P. Portaankorva-Koivisto, A. Laine & L. Näveri (Eds). *Current State of Research on Mathematical Beliefs XVIII: Proceedings of the MAVI-18 Conference, September 12-15, 2012, Helsinki, Finland.*(pp. 259–270). Helsinki: Unigrafia Oy.

Talmy, S. (2011) The interview as collaborative achievement: Interaction, identity, and ideology in speech event. *Applied linguistics 32(1)*, 25–42

Vetenskapsrådet. (2017). *God forskningssed* [Good research practice]. Vetenskapsrådets rapportserie 1:2001.

Wigfield, A., & Eccles, J. S. (2000). Expectancy-value theory of achievement motivation. *Contemporary Educational Psychology 25*, 68–81. https://doi.org/10.1006/ceps.1999.1015

II

Primary students' expressed emotions towards mathematics education

Martin Nyman

Department of Mathematics and Science Education, Stockholm University

A body of research highlights factors relating to students' emotions – towards themselves, the social environment, learning, and the subject itself – as being of pivotal importance for learning. This paper reports on a study where students in grades two and five were interviewed about their experiences with mathematics, especially focusing on expressed emotions. Using a combination of deductive frameworks and an inductive search, nuances in students' expressed emotions were revealed, with tentative results indicating that issues of control are significantly important and that boredom conceals emotionally important complexities.

Introduction

Previous research has indicated that emotion is an intrinsic part of every learning situation, including mathematics learning (e.g., Hannula, 2006; Radford, 2018; Ryan & Deci, 2000; Schukajlow et al., 2017). Emotions "simultaneously emerge from, and shape experience" (Liljedahl, 2014, p. 27) and thus, play a part in the individual's structuring of future action through the interrelationship between emotion, motive and action (Leont'ev, 2009).

It appears that student interest in, motivation for and engagement with mathematics is inversely proportional to years of schooling (e.g., Blomqvist et al., 2012; Hannula, 2006). However, available studies on affective factors like emotion primarily cover teenagers or adults (Dowker et al., 2019) and focus on mathematics anxiety in relation to solving tasks without explaining how these phenomena develop (Batchelor et al., 2019). Thus, there is reason to focus more on capturing the nuances within emotions, as well as the supposed link and interplay between different affective factors, to better understand the mechanisms behind student action. Therefore, the aim of this study is to explore student emotions in relation to mathematics and in particular nuances in expressed emotions by addressing the following research questions:

1) How do students express emotions in relation to mathematics and what are the characteristics within these expressed emotions?
2) How and by what mechanisms are students' emotions linked to expressed motives?

Background

The neurological way of describing emotions is to view them as bioregulatory reactions made up of chemical and neural responses that the brain produces, and that this production is performed automatically and in steps (Damasio, 2004). The initiation of the emotion is followed by biochemical changes in the body and brain and finally the emotion is made conscious, resulting in the person "feeling the emotion". This mechanism places emotion in the middle of the physiology-psychology divide as the mechanism that connects the two (Damasio, 2004). Understanding this connection is important for education. Since the aim of the present study is to explore nuances in students' expressed emotions, Schirmer's (2015) operationalisation of the concept is used. She defines emotion as "conscious or unconscious mental states elicited by events that we appraise as relevant for our needs and that motivate behaviours to fulfil these needs" (Schirmer, 2015, p. 26).

The division between emotion and feeling has been explored and discussed for mathematics education purposes by other researchers (e.g., Sumpter, 2019). One conclusion being that emotions include both bodily experiences and, sometimes but not always, a cognitive interpretation and/or expression of these experiences. Another conclusion is that the analysis of emotional nuance must expand beyond a positive-negative dichotomy.

Further, a body of research in the field of psychology has tried to establish whether the number of different emotional sensations are infinite or limited to a finite set of basic emotions (Shirmer, 2015). The theoretical starting point for this study comes from Löwheim (2011). He advocates a finite set of eight emotions, making the neurologically grounded argument that this figure represents the maximum number of configurations one can derive from the three synaptic amino acids involved in the process of producing sensations in the brain. These eight emotions are: excitement, joy, surprise, distress, fear, shame, disgust and anger. Though the number is fixed, the individual experience of emotional sensations may be more varied, since sensations occur to various extents and can be more or less intense. Contrasting this fixed number, Lewis (2013) presents a different – and larger – set of emotions at play in educational situations. He also discusses an interrelation between the concepts emotion and motivation. Theories for describing this interrelation are important for understanding the mechanisms for student action.

To further understand the role of emotion in student action in relation to teaching, it is important to acknowledge the different positions of the aquisitionist and participationist paradigms respectively (Liljedahl, 2014). The acquisitionist paradigm implicitly treats emotion as a psychological phenomenon that is a reaction to (interpretation of) the individual's experience and which regulates their future actions. In the participationist paradigm, the role of emotion is intertwined with the individual's actions through their motives which in turn

are created by their needs. Since motives, hence also needs, can be unknown to the individual, the emotions work as regulators of actions by feeding back the fulfilment of the individual's needs. As a consequence of the causal chain – needs-motives-action – the importance of motivation is significantly reduced in the participationist paradigm (e.g., Arievitch, 2017). The individual is understood not required to have motivation (i.e., pleasure, salary or grades) in order to do something, instead they are considered to have a need (personal, social or other), which creates a motive for action (Leont'ev, 2009). Independent of the paradigm there is still a need for expanding the multitude of affectively relevant dimensions even further. One attempt is made by Hannula (2006, 2012), who combines eight dimensions, grouped together in three dimensions on the sides of a cube – emotion, cognition and motivation make up the first side of the cube; psychological, physiological and social the second; and the two temporal dimensions state and trait the third side.

Looking at empirical studies conducted within the Swedish context, one indication reported by Blomqvist et al. (2012) is that students' emotional dispositions turn from positive to negative around the age of nine; but, due to methodological constraints, the study neither discusses nuances within the group positive versus negative, nor potential causes for these emotions. Another indication is the connection between emotion and motivation (Nyman & Sumpter, 2019) and between emotion and achievement (Palmér & van Bommel, 2018). These studies, conducted with youngsters ranging from six to 18, imply an interconnection between emotion and other affective and cognitive factors that is stable and established early. Karlsson (2019) discusses the issue of anxiety towards mathematics expressed by poorly achieving students and points to a strong social link to negative emotions, but also suggests that emotional sensations can be either the cause or the effect of a situation. These examples depict emotion as a concept both interconnected with other psychological and physiological constructs as well having both an inhibitory and a promotive function (see also, e.g., Hannula, 2015; Dowker et al., 2019).

Methods

Since the aim of this study involves capturing nuances in student expressions, semi-structured interviews were chosen as the method for data collection. This method combines providing the respondents freedom to elaborate on their thoughts with the structured format of a questionnaire. It also allows the interviewer to pose follow-up and clarifying questions and for the respondent to do the same. The interview guide was based on a seven-item questionnaire instrument developed by Dahlgren Johansson et al. (2010) and later also used in a study by Blomqvist et al. (2012). The questionnaire combined closed items using a four-step-based likert scale in the form of happy and sad faces, with

free-text items and a drawing task at the end. In the present paper, focusing on emotion, questions 1, 2 and 5: “How do you like math?”, “How do you feel before a math lesson?” and “How do you feel when you do math?”, respectively, will be discussed. All 19 interviews were conducted by the author with the interviewees face-to-face, one-on-one, in a room near the classroom during lesson time. Each interview was audio-recorded and lasted between 20 and 30 minutes.

Participants

Data were collected at three schools in an urban area, for convenience and to remain within the same municipality. One inner city and two suburban schools, located in opposite areas in the the municipality were chosen. Students between the ages of eight and 11 in grades two and five, respectively, were chosen to enable comparisons with the previous studies (Dahlgren Johansson et al. (2010) and Blomqvist et al. (2012)). Due to the respondents relative youth, it was necessary to spend some time with each class prior to data collection, and thus, for practical reasons, the number of participating schools had to be limited to three. Ethics considerations stipulated by the Swedish Research Council through Codex (Vetenskapsrådet, 2017) were followed, so every participant had written parental consent and were informed that participation was voluntary and that they could stop the interview without reason at any time. The aim of this study was to capture general nuances in students’ emotions towards mathematics rather than emotions related to extremely high or low levels of achievement, while the number of interviews had to be limited. Therefore, on the day of the interviews, the teachers were asked to pick out, from among the volunteering pupils, individuals that they considered to be neither extremely proficient in nor having serious difficulties with mathematics. This was done hoping to reach representatives from the presumably large and often self-sufficient group of students that “ just go about their business” during mathematics lessons. The teachers were also asked to consider pupils who would manage the interview situation comfortably without feeling stressed or uneasy. In total, 19 pupils were interviewed – 10 in grade two and nine in grade five.

Analysis

Prior to analysis, the interviews were transcribed by the author. The data were transcribed verbatim, including non-verbal communication like exclamations and extended pauses. Questions number 1 (“How do you like maths?”), 2 (“How do you feel before a maths lesson?”) and 5 (“How do you feel when you do maths?”) were those that explicitly framed emotions or feelings. Therefore, as a first analytical step, the transcripts were marked where the responses to the selected questions appeared. In addition, a second reading of the transcripts was made, looking for instances where respondents had made additional references

to emotions or feelings in the exchanges that were the result of follow-up on questions other than the selected three. The first step in the subsequent analysis was then carried out and the instances were coded Positive or Negative with utterances like "I like math when … " and "It makes me stressed when … ", respectively. An instance was coded Neutral when the respondent referred to factors that were emotionally neutral, such as hunger. The second step used Löwheim's (2011) theory of basic emotions to divide the three initial categories into sub-categories. Finding these categories involved paying close attention to the words or expressions used by the respondents. For example, "I feel relaxed" instances were coded under the subtheme Relief, while "fun", "like", "happy" were coded under the subtheme Joy. This example also illustrates a decisive difference in emotional strength or intensity where "happy" is considered to be stronger than "like".

In order to expand the analysis beyond the descriptive, a framework for analysing data by the types of motives or justifications in which respondents framed their responses was adopted as a third step. This framework was developed by Hannula (2012) and describes eight themes, four of which (cognitive, motivation, social and physiological) could be found in the data, while one is the construct which is the focus of this article – emotion. Here the coding focused on the specific ways the respondents expressed their experiences. For example, "I'm challenged by it" connects to the theme Motivation, whereas "I don't want my friends to laugh" connects to the theme Social. This example also highlights that each theme ranges over positive as well as negative emotions. The mapping of categories over Hannula's framework resulted in a number of responses, for example, the category Content carried properties that were inconsistent with any of Hannula's themes. This called for a fourth analytic step where the remaining responses were weighed against each other and the rest of the responses. This aimed to discern additional themes that captured the same level of explanatory dimension that the rest of the themes did. This step was an inductive search for similarities, inspired by the approach of thematic analysis (Braun and Clarke, 2006).

Results

The results are summarised in two tables – Table 1, followed by the analysis focusing on the first research question, and Table 2, building on this and focusing on dimensions of the second research question.

Table 1 shows the different emotional themes developed from data and an example for each theme. The right hand column shows the eight basic emotions listed in the theory section.

Table 1

Excerpts and themes mapped against constructs from theory of Basic emotions

Excerpt	Expressed emotions	Basic emotions
It's fun and exciting	Joy	Excitement
It feels like you're on top of things	Contentment	Joy
It feels safe having a kind teacher that helps you if you need	Relief	—
—	—	Surprise
Sometimes it's a bit tiresome	Discontentment	Distress
You get stressed about whether it's correct or not	Stress	Fear
I'm ashamed to ask	Shame	Shame
—	—	Disgust
—	—	Anger

Note. Basic emotions see Lövheim (2012).

As Table 1 shows, there are discrepancies between the themes derived from the expressions in the data and the theoretical constructs. The most salient of these discrepancies is the theme Relief. Students clearly expressed having feelings of relief in relation to managing an activity or coping with their tasks on time. This approaches being something of a double negative – the responses are positively loaded but the argument is based on the absence of something negative. The emotion Distress manifests primarily as discontent; however, the boundary with stress is not crystal clear; in fact, the emotion Fear manifests as something closer to stress. It seems that fear of not being able to solve problems, answering questions from the teacher, etc, causes stress. Looking at the rest of the themes, we see that full strength of the emotions are generally not expressed both, either on the positive or the negative side. And, the strongest emotions, especially the negative, are not present in the data.

Further, Table 2 summarises how expressed motives are distributed over the expressed emotions listed in Table 1. In the left column the expressed emotions are listed and the table shows the different ways these emotions are justified by the respondents, structured under five themes. The results are discussed with a focus on qualitative differences between categories and themes even though number of instances is presented (in brackets) to provide an overview of the relative frequencies between different themes.

Among the Positive emotions, the first subtheme is Joy where Process and Position are closely related and both link to Cognitive dimensions but differ in relation to outcome, whereas Challenge links to Motivational dimensions. In all three categories, respondents express enjoyment or happiness originating in the cognitive development they feel in relation to mathematics. When an utterance expresses this sense of development and cognitive expansion as being sufficient in themselves it is coded Process. When this shifts more towards the result of the work, which shows a linkage to the person appreciating their moving position on a "mathematical ability scale" this is coded Position.

Table 2

Students' expressed motivations for emotions experienced in relation to mathematics education. Number of instances within brackets.

EMOTION	Cognitive	Motivational	Technical	Personal (relates to self)	Social
POSITIVE (91)					
Joy (34)	Process (7) Position (8)	Challenge (19)			
Contentment (15)			Situation (14)	Autonomy (1)	
Relief (20)			Control (20)		
NEGATIVE (64)					
Discontentment (1)		Boredom (1)			
Stress (52)			LC Temporal (7)	LC Personal (45)	
Shame (11)				Personality (8)	Social (3)

Note. LC = Lack of Control.

Content is emotionally weaker than joy – positive, but with less intense sensations. The two concepts in this category both frame the working situation but for different reasons:

> Tania: Solving the maths tasks, and talking to your friend, sometimes we can /…/ talk a bit about maths and I think that's fun.
>
> Matteus: You can count in your own way. No one says you have to write this way or that.

The Situation theme, illustrated by Tania, is linked to technical issues around the working situation such as where to sit, being allowed to listen to music, and similar expressions of mathematics being a safe and comfortable activity. The Autonomy theme is also linked to the working situation, however, here the positive feelings are associated with the possibility of doing it "in your own way" as Matteus puts it. The Control theme also links to the working situation, but the feeling is expressed as the relief of being able, knowing how to do the work. Matteus again:

> Matteus: I feel calm, I can work at my own pace, not anybody else's pace. And it's not a contest about being first and so on.

Comparing the positive subthemes with each other indicates two main sources of emotion – one related to inherent mathematical properties and another to factors outside the mathematical content. And even though the emotional subcategories can be placed along an intensity continuum the different motive themes do not appear to form any similar simple pattern.

Before turning to the negative emotions, we see that the total number of positive responses is considerably larger than the sum of the different categorised positive responses (91 versus 69), and that negative responses have no such "overflow". Thus, there is a difference between how positive and negative

emotions are expressed. The 22 positive, non-categorised responses contain those saying "Good", without any further comments. This is not the case among the negative responses, which are always motivated. Among the negative responses, we see that the Boredom theme only has one instance, which calls for caution in drawing conclusions – Chris (grade 2) takes an unusually long time to answer the question, "How do you feel before a maths lesson?", but eventually chooses the happy-face card, however, with the following remark:

> Chris: Because it's a bit, like, I get tired from math but when I know we are going to do fun pages or something that perhaps is fun then it's number one [the happiest-face card].

Chris goes on to explain that he likes "when it gets difficult also" because "it ain't fun when you have really easy stuff, like two plus two and stuff". After this, the interviewer asks Chris what it is that makes him tired:

> Chris: It's when you must do super many pages, /…/ all the way to, like, page 80 when you're on page 65, then my arm gets really tired.

Chris describes mathematics as being fun; however, for it to be fun, some level of challenge (i.e. working on non-routine tasks) is necessary. In the light of this, Chris' tiredness is interpreted as negative and categorised as Discontent. Chris never explicitly says maths is boring or uses explicitly negative words to describe it. However, adding other instances where students generally describe "zoning out", the negative valence as well as its strength is clear, for example, Marcus: "Because when I start working in the maths book it gets a little messy /…/ so I can't concentrate because I just look at other stuff".

Among the other negative subthemes, Stress is the most commonly expressed emotion and its motive is related to lack of control, divided between a Temporal and a Personal subcategory (LC_T and LC_P, respectively). Not having enough time to finish tasks during an exam or keeping up with the pace of the class connects the emotion to temporal factors. The superficially similar category of personal lack of control (LC_P), is probably a more disadvantageous emotion because it links to issues of self-efficacy, self-control and potentially also self-worth. Chris' introspection is a typical LC_P-coded response:

> Chris: Like, "Oh no, I don't know how to do it!" and I bet it'll take a long time to learn.

Even though this kind of data are too small to draw quantitative conclusions from it, it is worth noting that this group is the largest among the negative responses, in fact the largest altogether.

Looking further at Shame, the results signal that the social dimension can come from feeling ashamed in relation to others, or from being outside a group. The category splits into two:

> Vera: It feels really difficult because my friend sitting next to me is quite fast and so then I feel a bit stressed – that I'm quite far behind then.

Vera describes a shameful feeling, however, not connected to herself or her abilities, but her concern is that she will be left behind, that she will literally be on her own. The decision is therefore to code this type of instance as Sociality. Now compare this to what Jane (grade 5) is saying:

> Jane: Sometimes, some say, like, "that one is really easy" – just blurts it out. Then I feel ashamed for asking. Because it feels like they might say that I'm bad at maths.

Jane's comment has the same social dimension, but this feeling originates from beliefs of self-worth. Therefore, this different type of shame-connected motive is coded Personality. Thus, a hierarchical structure seems to exist between the concepts: Personality–Sociality–LC_P–LC_T, in descending order of emotional strength and separated in levels of interaction.

Discussion

Given the important role emotion plays in learning situations (e.g., Hannula, 2012; Radford, 2018) and that emotion reciprocally both shapes and is shaped by a person's experience of education (Liljedahl, 2014), this paper contributes to describing and analysing these connections and to revealing some of the nuances within emotion and other affective constructs. This is relevant since emotion is a major ingredient in the individual's motives for engaging in education, and is therefore linked to achievement (Leont'ev, 2009; Schukajlow et al., 2017). When discussing the results, it is important to bear in mind that since this study is a small-scale interview study its primary contribution lies in offering avenues for continued research . However, from a methodological perspective, it is interesting to note that eliciting responses using likert-scale "faces" turned out to be more fruitful than anticipated when it prompted respondents to reflect on situations they associated with happy and sad faces, respectively. This resulted in every respondent contributing very rich data. Thus, the findings of this study can be sen as a qualitative continuation of the more quantitative studies by Blomqvist et al. (2012) and Dahlgren Johansson et al. (2010).

In relation to RQ 1 the results indicate that students are indeed emotional about mathematics, and Table 1 contains a summary of the ways these emotions are expressed. Looking at the characteristics and comparing the data with Löwheim's (2011) basic emotions we see that both positively and negatively loaded emotions are represented: joy, excitement, shame and distress. Factors of fear and distress manifest themselves more like stress and discontent, respectively, and occur both in test/exam situations and during ordinary lessons. These

factors are linked to personal or social dimensions in line with what others have reported (e.g., Karlsson, 2019; Samuelsson, 2011). Even though the strongest negative emotions of anger and disgust are not explicitly present in the data, it is a strong possibility that these emotions, together with fear, are all expressed as stress. This suspicion implies the need for further research. However, instances of "weaker" responses that are clearly emotional in nature are also present, like this feeling of relief expressed by Matteus: "I feel calm, can work at my own pace". In conclusion, even though considering feelings as conscious and emotion as unconscious (cf. Damasio, 2004; Schirmer, 2015) implies that respondents express thoughts about feelings rather than emotions, as educators we need to take these emotion-related experiences (both strong and weak) into account.

In relation to RQ 2, the ways students justify emotions, shown in Table 2, can be understood as a starting point for understanding the mechanisms connecting emotion to other affective concepts. For example, it seems that issues of motivation emerge when students are asked to describe emotion, and vice versa. This is parallel to the interplay between motivation and emotion reported by Nyman and Sumpter (2019), and Hannula (2012), where Motivation and Cognition, as well as social dimensions, are part of his model. Further, the Personal dimension appears to be very similar to Samuelsson's "self-concept" (2011). However, the Technical factors, contributing to negative as well as positive emotions, do not seem to be part of any previous theory. Considering these points and the limited size of this study, further research in this field is required – preferably using a participatory methodology – to capture the fleeting emotion-connected dimensions occurring momentarily during lessons. When looking at implications for teaching, the participationist perspective's connection (e.g., Arievitch, 2017) to action, through student emotions and motives, places the power to influence students more in the hands of the teacher – through well planned teaching – than a more psychological and individualistic view on emotion and affect does. The design of this type of teaching should also be the subject of further research.

One additional, and peculiar detail from this study is the absence of instances of the emotion Surprise. Shirmer's (2015) definition of emotion rests on the concepts of appraisal and need, and implicitly connects emotion to motive (e.g., Leont'ev, 2009). Inspired by Liljedahl's research on the "AHA!-experience" (2005), and since surprise can be connected to a motive for understanding (the reason for one's surprise), implies that surprise is a dimension of learning situations that could be made fruitful in order to enhance student learning. Thus, potentially a starting point for further research.

References

Arievitch, I. M. (2017). *Beyond the brain. An agentive activity perspective on mind, development and learning.* Sense Publishers.

Batchelor, S., Torbeyns, J., & Verschaffel, L. (2019). Affect and mathematics in young children: An introduction. *Educational Studies in Mathematics, 100*(3), 201–209. https://doi.org/10.1007/s10649-018-9864-x

Blomqvist, A., Elamari, U., & Sumpter, L. (2012). Grade 2 and Grade 5 students' conceptions about mathematics and mathematics education. In G.H. Gunnarsdottir, F. Hreinsdottir, G. Palsdottir, M. Hannula, M. Hannula-Sormunen, E. Jablonka, U. Jankvist, A. Ryve, P. Valero & K. Wæge (Eds.), *Proceedings of NORMA 11: The sixth Nordic conference on mathematics education* (pp. 187–196). University of Iceland Press.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), (pp. 77–101). doi: https://doi.org/10.1191/1478088706qp063oa

Dahlgren Johansson, A. & Sumpter, L. (2010). Children's conceptions about mathematics and mathematics education. In K. Kislenko (Ed.) *Current state of research on mathematical* beliefs XVI (pp. 77-88). OÜ Vali Press.

Damasio, A. (2004). Emotions and Feelings. In A. Manstead, N. Frijda, & A. Fischer (Eds.), *Feelings and Emotions: The Amsterdam Symposium (Studies in Emotion and Social Interaction*, pp. 49–57). Cambridge University Press. doi:10.1017/CBO9780511806582.004

Dowker, A., Cheriton, O., Horton, R., & Mark, W. (2019). Relationships between attitudes and performance in young children's mathematics. *Educational Studies in Mathematics, 100*(3), 211–230. https://doi.org/10.1007/s10649-019-9880-5

Hannula, M. S. (2012). Exploring new dimensions of mathematics-related affect: Embodied and social theories. *Research in Mathematics Education, 14*(2), 137–161. doi: https://doi-org.ezp.sub.su.se/10.1080/14794802.2012.694281

Hannula, M. S. (2015). Emotions in problem solving. In S. J. Cho (Ed.). *Selected regular lectures from the 12th international congress on mathematical education,* (pp. 269–288). doi: 10.1007/978-3-319-17187-6_16

Hannula, M. S. (2006). Motivation in Mathematics: Goals Reflected in Emotions. *Educational Studies in Mathematics, 63*(2), 165. https://doi-org.ezp.sub.su.se/10.1007/s10649-005-9019-8

Karlsson, I. (2019). *Elever i matematiksvårigheter: Lärare och elever om låga prestationer i matematik* [Doctoral dissertation, Lund University]. Lund Studies in Educational Sciences. www.ht.lu.se/serie/lses

Leont'ev, A. N. (2009). *Activity and Consciousness.* CC-SA: Marxist Internet archive.

Lewis, G. (2013). Emotion and disaffection with school mathematics. *Research in Mathematics Education*, *15*(1), 70–86. https://doi.org/10.1080/14794802.2012.756636

Liljedahl, P. (2014). Emotion as an orienting experience. In L. Sumpter (Ed.), *Proceedings of MAVI-20 Conference.* (pp. 21–32). Högskolan Dalarna.

Liljedahl, P. (2005) Mathematical discovery and affect: the effect of AHA! experiences on undergraduate mathematics students, *International Journal of*

Mathematical Education in Science and Technology, 36:2-3, 219-234, DOI: 10.1080/00207390412331316997

Löwheim, H. (2011). A new three-dimensional model for emotions and monoamine neurotransmitters. *Medical Hypotheses 78*, (pp. 341–348). doi:10.1016/j.mehy.2011.11.016

Nyman, M. & Sumpter, L. (2019). The issue of 'proudliness': Primary students' motivation towards mathematics. *LUMAT*, (2). https://doi.org/10.31129/LUMAT.7.2.331.

Palmér, H. & van Bommel, J. (2018). Young students' feelings towards problem-solving tasks: what does "success" imply?. In B. Rott, G. Törner, J. Peters-Desdemir, A. Möller, Safrudiannur, (Eds.). *Views and beliefs in mathematics education. The role of beliefs in the classroom,* (pp. 69–78). doi: https://doi.org/10.1007/978-3-030-01273-1

Radford, L. (2018). Teaching and learning (algebra or something else): Working together to make sense of similarities and differences between theories (and understanding oneself). Paper Presented at the AERA Symposium Dealing with Diverse Discourses (3D): Can We Build on Each Other's Research Contributions?, New York.

Ryan, R. M., & Deci, E. L. (2000). Intrinsic and Extrinsic Motivations: Classic Definitions and New Directions. *Contemporary Educational Psychology, 25*, (pp. 54–67). doi: 10.1006/ceps.1999.1020

Samuelsson, J. (2011). Development of self-regulated learning skills in mathematics in lower secondary school in Sweden. *Nordisk matematikkdidaktikk: Vol. 16:3*. Nordisk matematikkdidaktikk. http://urn.kb.se/resolve?urn=urn:nbn:se:liu:diva-74718

Schirmer, A. (2015). What is an emotion? Modern thoughts and concepts. *Emotion.* (pp. 42–70). Sage Publications. doi: http://dx.doi.org/10.4135/9781483378619.n4

Schukajlow, S., Rakoczy, K., & Pekrun, R. (2017). Emotions and motivation in mathematics education: Theoretical considerations and empirical contributions. Springer Science + Business Media.

Sumpter, L. (2019). Preschool educator' emotional directions towards mathematics. *International Journal of Science and Mathematics Education*, (1). https://doi.org/10.1007/s10763-019-10015-2

Vetenskapsrådet. (2017). *God forskningssed* [Good research practice]. Vetenskapsrådets rapportserie 1:2001.

III

Secondary students' expressed emotions and motivation towards mathematics

Martin Nyman

Department of Science and Mathematics Education, Stockholm University

Introduction

The importance of affective dimensions in learning is acknowledged in previous research (e.g., Hannula, 2019; Lewis, 2013; Schukajlow et al., 2017). Issues like emotion, belief, motivation and attitude are interrelated and influence each other (e.g., Hannula, 2012). They also have a strong influence on achievement and learning, and it is even suggested that variables such as attitude, motivation and self-concept are the dominant predictive factors for academic performance, more decisive than cognitive factors and socio-economical background (Suárez-Álvarez et al., 2014). Moreover, these factors seem especially important in mathematics education; for example, Hannula (2011) believes that mathematical thinking is intrinsically interwoven with emotion, and Barnes (2019) develops the notion further in relation to reasoning, one of the foundations of mathematics, when stating that, "[t]he emotions associated with reasoning are not simply a by-product of cognition that can be isolated and disregarded" (Barnes, 2019, p. 272). Thus, conducting studies and striving for detail to deeply understand students' affective and, especially, emotional relations to mathematics are relevant.

Research indicates that students' joy for and motivation towards mathematics drops during the compulsory school years (Blomqvist et al., 2012; Hannula, 2006). There are also differences between how boys and girls perceive their potential in mathematics and between their emotional attitudes towards mathematics (Frid et al., 2020). At the same time, researchers point to an asymmetry in researched ages where affect- and attitude-related issues are more well researched among upper secondary school students, university students and adults than at primary and lower secondary school levels (Dowker et al., 2019). Furthermore, research comparing students' affective variables over the complete age spectrum of compulsory schooling is limited. In addition, the cultural dimension in affective constructs (e.g., Arievitch, 2017; Radford, 2015) and differences between countries, school systems and technologies call for caution when drawing far-reaching conclusions from international comparisons of both research and student achievement.

This study being set in Sweden enables the analysis of a problematic link between motivation and mathematics known as the "educational-gender-equality paradox". This concept was coined and described by Stoet and Geary (2018) to describe the fact that the most gender equal countries within the OECD community produce the largest gender gaps in STEM fields in higher education. Stoet and Geary refer to Finland, then Norway and Sweden as leaders in this ostensible paradox. Comparisons of PISA scores reported for the measurements carried out 2018 (OECD, 2019) show gender differences in motivation scales and items related to future career choices to be above average in all three countries.

This paper discusses how students in grades eight and nine (aged 14 and 15, respectively) express their affective experiences related to mathematics and their experience of mathematical praxis by answering the following research questions:

1. How do students express their feelings about doing mathematics?
2. How do students express their motivation for doing mathematics?
3. How do motivation and emotion interrelate?

Further, to better understand the previously reported gender differences in the STEM-field potentially due to issues of emotion and motivation, a fourth research question is posed:

4. What gender differences can be found in relation to questions 1 and 2?

Background

Taking Hannula's (2012) framework for affect as a starting point for this paper offers a well described structure of theoretical concepts, among which *emotion* and *motivation* are the focus. In addition, student *conceptions* – about mathematics and mathematics education – will be touched upon.

In biochemical research emotion is described as bioregulatory reactions to internal or external inputs whose onset is often not conscious (Hannula, 2012). A description from Damasio (2004) offers a theory to differentiate between emotion and feeling: An emotion is the initial and nonconscious response to an internal or external stimulus. The feeling is then the conscious product of that stimulus. In this study, emotion is defined as, "conscious or unconscious mental states elicited by events that we appraise as relevant for our needs and that motivate behaviours to fulfil these needs" (Schirmer, 2015, p. 26).

Motivation, considered to be the individual's driving force that compels them to action, can be more precisely defined as "the process whereby goal-directed activity is instigated and sustained" (Schunk et al., 2010, p. 4). In this study, motivational expressions are divided between intrinsic and extrinsic (Ryan & Deci, 2000), where an intrinsically motivated person is driven by the pure satisfaction

of doing something and extrinsic driving forces are linked to external factors, which in school settings are typically achieving grades or avoiding scolding.

Both motivation and emotion reciprocally affect and are affected by a person's conception about a certain activity (Hannula, 2012; Beswick, 2018). According to Philipp (2007), affect consists of emotion, attitudes, and beliefs and all three are to varying degrees cognitive in nature. Further, conception is defined as "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences" (Philipp, 2007, p. 259). In this study, students' conceptions consist of their emotional experiences of mathematics education, combined with their personal interpretations of influential persons' expressed opinions of mathematics, and the role of mathematics in society. These influences form stable mental images in an individual regarding their relations with mathematics education (e.g., Samuelsson, 2011). Conception thus inevitably has both cognitive and emotional dimensions.

A questionnaire survey among pupils in seven Swedish classes in grades two and five (aged eight and 11, respectively) was carried out by Blomqvist et al. (2012). The purpose was to investigate differences in students' conceptions about mathematics and mathematics education between these two grades. The background to the study was indications from international research as well as national assessments carried out by the Swedish National Agency for Education, that students' interest in mathematics peaked in grade five and dramatically dropped during grade eight. The study by Blomqvist et al. (2012) indeed confirmed the suspected drop and even suggested that it occurs considerably earlier, somewhere between grades two and five but could say little about the exact age when this shift occurred, and even less about how it develops in higher grades. Neither did it investigate potential gender differences.

Continuing with the theme of reduced interest in mathematics over time, a study by Di Martino (2019) showed how pre-schoolers had a fundamentally different – and much more desirable – approach to mathematics than their second- to fifth-grade companions. Di Martino's study focused on problems and problem-solving, and paints a solemn picture:

> /…/ in a certain sense, it seems that educational experience – and, in particular, the exposition to mathematical problems in primary school – has a negative effect on students' vision of problems, but also on their self-perception and emotional disposition towards mathematical problems (i.e., on students' attitude towards mathematical problems). (Di Martino, 2019, p. 305)

This is yet another example of the connection and interdependence between emotion and motivation discussed in several studies, as previously mentioned. One remaining question, however, is the extent to which these phenomena reciprocally predict each other. A Swiss study by Sutter-Brandenberger et al. (2018) confirms this connection but dismisses fundamental bidirectionality; for

example, intrinsic motivation negatively predicted boredom in seventh grade students but not conversely, and no significant correlation between these phenomena could be found in eighth grade.

Gender issues in affect towards mathematics education

Conceptions of mathematics, mathematics affordances, and one's own possibility to access mathematics and thereby gain mathematical knowledge, seem to differ between genders, both at a collective level (Frid et al., 2020; Sumpter, 2012) and when shaping individual choices for higher education (Stoet & Geary, 2018). However, the term gender "is both ambiguous and ambitious" (Nielsen, 2017, p. 264) and can refer to (at least) three different things: it can refer to simple distinctions between male and female attributes associated with biological sex; it can refer to a difference between men and women in the distribution of some entity (for example, the distribution of hormone levels or physical strength, but also behaviour, attitude, influence or money); or gender may refer to a discourse. Gender discourses are culturally and historically rooted and intertwined with other discourses such as race, ethnicity and social class. As such, gender discourses reciprocally affect and are affected by our understanding and interpretation of society and the world. Which of these three gender dimensions is the foundation for interpretations based on gender as a concept is often not explicit, defined, or even acknowledged. Nielsen calls them the distinctive, the distributive and the discursive interpretation of gender, and laments that the three are often mixed up (Nielsen, 2017).

The distributive dimensions of gender are the focus of this study which looks for differences between the emotional and motivational dispositions of girls and boys towards mathematics.

For educational purposes, the term "doing gender" (e.g., Bjerrum Nielsen, 2003; Wedege, 2011) is important when considering implications for teaching and overall interaction in school settings. Doing gender refers to social situations constructs non-explicit gender stereotypes, people "actively 'do' gender in ways that are meaningful to them culturally as well as personally" (Bjerrum Nielsen, 2003). And the purpose of analysis is to "demonstrate how gender is done, how it is negotiated, maintained or eventually changed" (Nielsen, 2017, p. 27). Thus, gender differences are not reduced by schooling, they are produced there. Accordingly, studies indicate that gender stereotypes may appear more fixed than the individual's view of themself (Frid et al., 2020; McGeown & Warhurst, 2020), and that differences between the sexes concerning attitude and self-belief in relation to mathematics and science increase with age (McGeown & Warhurst, 2020).

Method

This paper reports on an independent continuation of the questionnaire study conducted by Blomqvist et al. (2012) consisting of a seven-item questionnaire distributed to pupils in grades two and five (aged 8 and 11, respectively). The exact same instrument was used, but here the respondents were in grades eight and nine (aged 14 and 15, respectively).

Participants

Aiming to capture the diversity in socio-economic backgrounds in the participating schools, I used the index published by the Swedish National Agency for Education (Skolverket, 2018) to search for schools that were also practical to visit. This purposive sampling resulted in six participating schools with an index span of 152 units on an index scale ranging from 20 to 400[1]. At the volunteering schools, mathematics teachers responsible for grades eight and/or nine were asked to participate. I visited each participating class while the questionnaires were being administered during a class taught by the students' regular mathematics teacher and most often it was a mathematics lesson. Following the ethics guidelines stipulated in Codex (Vetenskapsrådet, 2017), prior to administering the questionnaires each student had secured written (parental, if necessary) consent and were informed that participation was voluntary.

In total, 222 students participated – 78 in grade eight and 144 in grade nine: 110 girls, 109 boys. Three individuals left the gender choice blank and these three respondents were subsequently not included in the gender comparisons.

Instrument

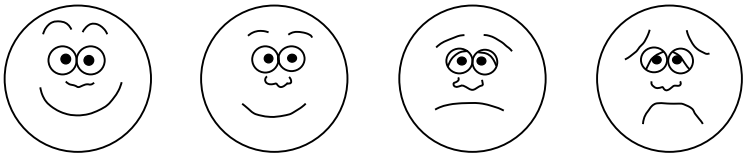
The questionnaire consisted of seven items, that can be crudely divided into three methodologically different parts:

Part 1: Quantitative. Items 1–3: "What do you think about maths?"; "How do you feel before a maths lesson?"; and "How do you feel before a Swedish [mother tongue] lesson?" Respondents chose from a four-step Likert scale in the form of happy and sad faces, see Figure 1. Because this study constitutes the continuation of a previous study, an unaltered version of the instrument was used and my somewhat older respondents (aged 14–15) were given the same Likert scale as originally designed for eight-year-olds. Explaining this to the respondents was one of the reasons I needed to be present during data collection. Another reason for my presence was that respondents can find this type of response window constraining and either leave an item unanswered or choose more than one alternative, accompanied sometimes with a comment. Therefore, I wanted to be present during data collection to explain the connection to previous studies as well as explain to the respondents that they were entitled to respond beyond the scope of the questionnaire format if they wanted.

[1] Higher indexes exist in schools specialising in educating newly arrived immigrants.

Figure 1

Likert scale used in items 1–3



Part 2: Quantitative/Qualitative. Items 4–6: Three open-ended questions where the responses were subsequently deductively and inductively placed in categories (see Analysis section, below). The questions items were: "Why do you do maths?", focusing on motivational issues; "How do you feel when you do maths?", focusing on emotional issues; and "What do you do when you do maths?", focusing on descriptions of what type of action or activity is associated with mathematics.

Part 3: Qualitative. Item 7: The question, "(p)lease make a drawing of yourself doing maths", affords respondents possibilities to show what takes place, who is involved, what tools are used and how it is done while doing mathematics, as well as to illustrate the emotions, communication and social interactions present during these mathematics activities.

In previous work leading up to this study, I had used the same questionnaire instrument as an interview guide. My experience from this undertaking was that respondents, for various reasons, found these questions unfamiliar, even strange and difficult to answer, so I wanted to be present during data collection. The previous respondents' hesitancy was often linked to a belief that a particular response was expected; so, during this study, I gently urged respondents to search their memory for personal experiences of mathematics without trying to "please" anyone or produce any specific or "correct" answer.

Analysis

Even though both emotion and motivation can be described with a richness of nuance, in a quantitative research setting such as this, these constructs can also be "reduced" to a valence. In previous research, emotion is understood as emotional orientation, meaning it is either positive or negative (Blomqvist et al., 2012), and motivation is understood in terms of its intrinsic or extrinsic origin (Blomqvist et al., 2012; Nyman & Sumpter, 2019; Ryan & Deci, 2000). In this paper, research questions one and two are intended to continue this research by examining older students' experiences.

A response to any of the first three items – with Likert responses – was coded positive when either one of the happy faces was marked, and accordingly negative when either of the sad faces was selected. A response containing

one face of each valence was coded neutral; the alternative would have been to disregard such responses.

The analysis of the motivational dimension in Item 4 was made deductively in four categories – first, two pure, single-construct categories: intrinsic, referring to enthusiasm or wanting to do something, and extrinsic, referring to coercion or having to do something (Nyman & Sumpter, 2019; Ryan & Deci, 2000). Further, one mixed category, capturing the responses that expressed a mix of both enthusiasm and coercion, like "because it's fun and I have to". And lastly, a neutral category for instances with no explicit references to motivation of any kind, like "to learn more".

Item 5, which asked about feelings during a mathematics activity, much resembled the first three and was coded with the same categories: positive when positive emotions dominated, or when responses contained dimensions of agency (Lange, 2009; Radford, 2018); negative when negative emotions dominated, or when emotional ambivalence leaned towards the negative, for example, "sometimes it's okay but often I'm a bit stressed"; and neutral when two opposite emotional valences were expressed or when the response implied indifference.

Items 6 and 7 both link to descriptions of mathematics activity, and both were analysed following Blomqvist et al. (2012), where a set of elements – either depicted or described – construed the different categories. In both cases, due to a large proportion of data falling outside the categories, an *Other* and an *Unknown* category were needed. The data in these categories were analysed using inductive methods, inspired by the approach of thematic analysis (Braun & Clarke, 2006). This phase was an inductive search for similarities and aimed to discern additional themes that captured the same level of explanation that the rest of the themes did. In addition to its activity dimensions, Item 7 could also be analysed from an affective perspective, looking at what type of thoughts or emotions the pictures contained. In this respect, the analysis was done following previous emotion-loaded items, in positive, negative or neutral categories. Looking for emotional dimensions made it necessary to include an *Unknown* category for illustrations without emotional information, for example, when the picture did not show faces or only showed details from a person's workplace, such as books, ruler, pencil, calculator, etc. It could be argued that these images should be considered neutral, based on the interpretation that they show emotionally neutral situations. However, it could also be argued that these images should be considered negative, or at least not positive, since they show an emotionally sterile situation. Remedying this ambiguity, the choice was to simply label them *Unknown*. The illustrations often also contained text and other symbols; thus, the analysis was an integration of the depicted persons' faces and other expressions together with text and other details in their pictures. For example, the text "I'm bored" accompanying an otherwise neutral face led to a negative interpretation, or a big, shining lightbulb over a person's head, led to a positive interpretation.

After coding, relative frequencies for the different themes (Negative, Positive, Intrinsic, Extrinsic, etc.) were decided for each of the respondent groups: grade eight students, grade nine students, girls and boys, respectively. Coding for nominal categories led to a restricted number of available statistical measurements. Enabling comparison between more than two categories and keeping the number of methods to a minimum led to using χ^2 as a measure.

As a first step, all significances were calculated using χ^2 with appropriate degrees of freedom considered for frequencies comparing grades. When this resulted in almost none of the differences showing significance, a second step was implemented, in the same way as the first, where frequencies comparing gender were decided. Generally, probabilities at 0.05 or lower were considered significant; however, in the tables, stronger significance levels are indicated as 0.01; 0.001, etc.

Results

Since the aim of this article is to discuss the way in which lower secondary students' affective experiences relate to mathematics praxis by looking at how they express emotion and motivation, the Results section will focus on the four questionnaire items framing these constructs. However, responses to other items will be included where relevant to strengthen or compare the results and thereby work as a backdrop for the results and later, the discussion.

Emotion

Items 2, 5 and 7 all frame emotion – Item 2 quantitatively, with a Likert response, and items 5 and 7 qualitatively, with free-text responses. Table 1 shows responses to these items distributed over age and gender.

We see in Table 1 that Item 2 generates a 55–45 relationship between positive and negative. Grade 8 students respond neutrally more often than their grade 9 companions, and respond negatively less often even though their positive scores are equally frequent. This 55–45 relationship between positive and negative is roughly mirrored between boys and girls.

Looking at Item 5, which also explicitly frames emotion, the positive category is remarkably similar in size to Item 2. However, the neutral category is considerably larger, with a typical neutral response being, "I really like working by myself in the workbook, but I get super stressed with tests and exams", or, "(d)ifferent. It depends on which math section"; the main reason being that the level of complexity in the individual's feelings towards mathematics education becomes visible when respondents are asked to use their own words. A gender comparison shows, in this case, that boys responded neutrally significantly more often than girls did, and the girls responded negatively significantly more often than the boys, even though both genders have almost identical positive scores.

Item 7 asked the respondents to illustrate what they do in mathematics. The pictures mainly contained references to making calculations and thinking about tasks in a mathematics textbook, sitting at a desk. Often a person was drawn doing these activities, occasionally with a peer or under the supervision of a teacher. In addition to these activities, the pictures illustrated expressions of emotion. This emotional dimension is what Table 1 shows. However, due to the focus on emotion, Table 1 does not include a column for the 38 percent of illustrations containing no personal information at all – for example, a calculator, or a person turning their back towards the viewer. In addition, the large neutral and unknown categories lead to relatively low scores on both positive and negative valences.

Table 1

Distribution of responses over the Emotional subscales in relation to item 2 ("How do you feel before a maths lesson?"), 5 ("How do you feel when you do maths?") and 7 ("Draw a picture of yourself when you do maths"). % (n).

Item	Respondent	Positive	Negative	Neutral	Total	*p*
2	Grade 8	55 (43)	37 (29)	8 (6)	100 (78)	
	Grade 9	55 (79)	44 (63)	1 (2)	100 (144)	> 0.5
	Girls	54 (59)	43 (47)	4 (4)	100 (110)	
	Boys	57 (62)	39 (43)	4 (4)	100 (109)	> 0.5
5	Grade 8	56 (44)	29 (23)	14 (11)	100 (78)	
	Grade 9	52 (75)	40 (57)	8 (12)	100 (144)	> 0.1
	Girls	53 (58)	42 (46)	5 (6)	100 (110)	
	Boys	54 (59)	30 (33)	16 (17)	100 (109)	< 0.05
7	Grade 8	54 (25)	15 (7)	30 (14)	59 (46)	
	Grade 9	48 (44)	32 (29)	20 (18)	63 (91)	> 0.05
	Girls	54 (38)	25 (18)	21 (15)	65 (71)	
	Boys	48 (30)	27 (17)	25 (16)	58 (63)	> 0.5

Note. Significances computed for grades and gender separately.

The gender differences in Item 7 are practically nil and point to positive scores being in line with items 2 and 5 but with considerably higher neutral scores. However, when comparing ages, positive and neutral scores drop, and negative scores go up from grade 8 to grade 9. These differences are significant at 0.01.

To summarise, it is interesting to compare the results from Item 2 with the results from Item 1 where students were asked: "What do you think about maths?" This question generated a summed distribution of 71% positive and 29% negative (n = 153 Positive, 62 Negative, 7 Neutral), indicating that students had a more positive general notion of the importance and use of mathematics than personal emotional experience from actually doing mathematics. The difference between items 1 and 2 is significant at 0.01.

A concept that straddles emotion and motivation is boredom. In this study, 18% of the respondents (the same frequency for boys and girls) explicitly mentioned being bored while doing mathematics. Among those are a few responses looking like a dictionary definition, like Fredrik, who said, "(i)t's easy and quite boring". However, more frequent are instances with a very different meaning, like Alice's comment that "(i)t's fun when you understand but otherwise boring and tiresome", or Caroline's, "I think it's hard and boring". It seems unlikely that the teacher would be able to remedy these three students' boredom in the same way.

Motivation

Table 2 shows the distribution of responses to (motivation) Item 4: "Why do you do maths?" The result is divided into four categories.

Table 2

Distribution of responses over the Motivational subscales in relation to item 4. % (n).

Respondent	Intrinsic	Mixed	Extrinsic	Neutral	Total	*p*
Grade 8	29 (23)	6 (5)	63 (49)	1 (1)	100 (78)	
Grade 9	22 (31)	8 (12)	66 (95)	4 (6)	100 (144)	> 0.1
Girls	18 (20)	12 (13)	70 (77)	0 (0)	100 (110)	
Boys	29 (32)	4 (4)	61 (66)	6 (7)	100 (109)	< 0.005

Note. Significances computed for grades and gender separately.

From Table 2 we see that an extrinsically motivated response is twice as common as an intrinsic response, but also that about ten percent of the responses are either mixed or neutral. A typical neutral-coded response, "(t)o learn more", points to the importance of understanding that a neutral response does not necessarily mean the person is unmotivated.

The results presented in Table 2 indicate small differences between ages, and irrespective of whether we compare two, three or all four categories, none of these differences are significant. But shifting the comparison to gender paints a slightly different picture. Girls and boys seem to differ substantially when asked about motivation for doing mathematics. These gender differences are significant ($p = 0.0015$) when considering all four categories. Further, the definition of the mixed category makes it possible to include mixed responses in the extrinsic category and thus separate purely intrinsic values from every other type of motivation. This interpretation renders an intrinsic–non-intrinsic ratio for girls slightly less than 1:5 and a corresponding ratio for boys just above 2:5. This difference has a significance level of $p = 0.026$. Thus, regardless of interpretation of varieties of extrinsic motivation or amount of extrinsic qualities in mixed responses, the results indicate that girls are less intrinsically motivated than boys.

The relation between emotion and motivation

When looking further at how motivation and emotion are interrelated, it seems that only motivation statements from the intrinsic or mixed category relate to emotion, and then only to positive emotions (see Table 3).

Table 3

Types of relation between Motivation category and Emotional valence in Motivation responses

Example from Item 4	Motivation category	Emotional valence
”Mostly I think it’s fun an challenging, so I just keep on /.../. ”	Intrinsic	Positive
”Because it’s fun and I have to.”	Mixed	Positive

Note. Item 4 = ”Why do you do maths?”

Since the question in Item 4 is about motivation, it cannot be expected that all responses reveal emotional valence. However, the ones that do are invariably connected to positive emotions and, as illustrated in Table 3, only to intrinsic or mixed motivation.

Looking further into the emotion-motivation interrelation, Table 4 shows the different existing variants of motivational connection to different emotion categories.

Table 4

Types of relation between Emotion category and Motivational connection in Emotion responses.

Example from Item 5	Emotion category	Motivational connection
”Not fun, but necessary.”	Negative	Extrinsic
”It’s important /.../.”	Neutral	Extrinsic
”Pretty motivated when I’m doing well.”	Positive	Extrinsic
”If it’s too easy it’s boring /.../ challenging tasks makes me focused.”	Positive	Instrinsic

Note. Item 5 = ”How do you feel when you do maths?”

Parallel to Item 4, not every statement from Item 5 is motivationally connected. However, the examples in Table 4 illustrate that emotion statements in all categories – negative, neutral, and positive – can be connected to extrinsic motivation, but positive emotions are exclusively connected to intrinsic motivation. Furthermore, the relation between positive emotion and intrinsic motivation seems reciprocal since the same connection is found both in Item 4 and Item 5. Even though not every statement contains this duality, the examples illustrate the need for considering motivation issues in connection with emotion and vice versa. Figure 2 is an illustration of these connections.

Figure 2

Model of dependence between Motivational and Emotional dimensions

Motivation ⟷	Emotion
Intrinsic	Positive
Mixed	Positive
Extrinsic	Positive Neutral Negative

The model in Figure 2 illustrates that positive emotions can be seen as a prerequisite for motivation of three different types, whereas both neutral and negative emotions are prerequisites only for extrinsic motivation. Present in Figure 2, although only implicitly, is the emotional feedbacking function that the motivational state has on the individual experience. This means that being intrinsically motivated generates positive emotions, as does extrinsic motivation, and that negative emotions are only generated when the individual is extrinsically motivated. The model thus also suggests a bidirectionality between the two concepts.

Discussion

In line with previous research (e.g., Nyman, in press; Takeuchi et al., 2016), mathematics is considered an important subject, both for further education and for life, sometimes despite personal emotional experiences of the subject being problematic. And girls significantly more often than boys express negative emotional experiences in relation to mathematics. The combined effect of factors influencing students' experiences are succinctly described by Edward in grade 9: "The math in itself is fun but pressure from grades and a 'non-pedagogical' teacher make it all suck."

Looking further into motivation, the results indicate that students generally express being extrinsically rather than intrinsically motivated. Due to their having more experience of both mathematics education and school in general (Barnes, 2019), there is reason to expect a shift towards a relatively higher degree of extrinsic motivation in the older students. Since the same instrument was used in the present study as in the Blomqvist et al. (2012) study, it is relevant to compare the results. Our findings confirm the previous result that extrinsic factors increase with age; Blomqvist et al. saw a transition from a 9:1 ratio in grade two, to almost equal in grade five, and the present study found a 3:7 ratio in the secondary grades. However, further research that samples a cohort covering

this entire range of ages is needed to confirm this trend, as well as to discern the various effects of this shift in motivation; and, even more crucially, to understand if such a change is benign or detrimental, both in relation to learning achievement and personality development. This becomes especially important since a further result from the present study, based on differentiating between genders, is that girls significantly more frequently express extrinsic motivation factors towards mathematics education than boys do (see also Frid et al., 2020). It is reasonable to believe that some explanation of the educational-gender-equality paradox (Stoet & Geary, 2018) could be gained from such investigations.

As others have also previously reported (e.g., Hannula, 2019), our results indicate a connection between emotion and motivation, suggesting not only that positive emotions are linked to both intrinsic and extrinsic motivation, as well as to a blend of the two, but also that negative emotions only link to extrinsic motivation. This type of connection between affect-related constructs and their potential bidirectionality, are discussed by Sutter-Brandenberger et al. (2018). Further research seems needed to better understand these issues and eventually describe the dynamics and mechanisms in the affective domain.

Implications

When students get older, extrinsic factors like grades and competition for attractive upper secondary schools seem to have an increasing impact on students' self-concept concerning mathematics (Samuelsson, 2011). However, even though the results show that the change is dramatic, this does not necessarily mean that the effect is dramatic. In other words, even though intrinsic motivation factors may appear more palatable, associated as they are with words like, "I love it" and "I really look forward to doing it", this does not automatically imply that extrinsic factors are bad and that teachers should strive to avoid them. Here, a potentially interesting opportunity for both teaching and continued research lies in designing and developing lessons that provide students with a motive for learning, based on feelings of need that go beyond momentary enthusiasm or the prospects of future prosperity. Since extrinsic factors are found to be more dominant in girls it is likely that they would benefit even more from a different, more motive-based (e.g., Arievitch, 2017) teaching than boys. This issue is further illustrated by two quotes from Item 5, connected to emotions during mathematics activities. First, the connection "difficult = good": "If it's challenging then I'm glad. If it's too easy, I'll be bored", then the connection "difficult = bad": "When it's difficult I'm angry and try again. When it's easy it's fun." These quotes highlight the connection between emotion and motivation. But more importantly, they challenge teachers to distinguish between two students who have seemingly similar experiences of mathematics being boring, but who have fundamentally different reasons.

This study found significant changes in students' emotional and motivational attitudes towards mathematics over time, echoing Hannula's (2019) study that discussed the origin of students' views and beliefs about mathematics in the first years of schooling and which asked: "How and when does mathematics become a source for anxiety or pride?" (p. 309).

And an additional question arises: If students' views change from believing that mathematics is interesting and, above all, important and "worthwhile", what do their views change *to*? Further qualitative research is needed to answer these questions to improve students' achievements and expand their knowledge. On a quantitative level, this study only cautiously points in a certain direction and more research will be needed to establish more precisely the velocity and dimensions of such changes during the compulsory school years.

References

Arievitch, I. M. (2017). Beyond the brain: An agentive activity perspective on mind, development, and learning. Sense Publishers.

Barnes, A. (2019). Perseverance in mathematical reasoning: The role of children's conative focus in the productive interplay between cognition and affect. *Research in Mathematics Education*, *21*(3), 271–294. https://doi.org/10.1080/14794802.2019.1590229

Beswick, K. (2018). Systems Perspectives on Mathematics Teachers' Beliefs: Illustration from Beliefs About Students. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.). *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 3–18). Umeå, Sweden: PME.

Bjerrum Nielsen, H. (2003). One of the boys? Doing gender in Scouting. Génève: World Organization of the Scout Movement.

Blomqvist, A., Elamari, U., & Sumpter, L. (2012). Grade 2 and grade 5 students' conceptions about mathematics and mathematics education. 187–196.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3(2)*, 77–101. https://doi.org/10.1191/1478088706qp063oa

Damasio, A. R. (2004). *Emotions and Feelings*. Feelings and Emotions: The Amsterdam Symposium. https://doi.org/10.1017/CBO9780511806582.004

Di Martino, P. (2019). Pupils' view of problems: The evolution from kindergarten to the end of primary school. *Educational Studies in Mathematics*, *100*(3), 291–307. https://doi.org/10.1007/s10649-018-9850-3

Dowker, A., Cheriton, O., Horton, R., & Mark, W. (2019). Relationships between attitudes and performance in young children's mathematics. *Educational Studies in Mathematics*, *100*(3), 211–230. https://doi.org/10.1007/s10649-019-9880-5

Frid, S., Nortvedt, G. & Sumpter, L. (In press). Who is best in mathematics? Grade nine students' attitudes about boys, girls and mathematics.

Hannula, M. S. (2012). Exploring new dimensions of mathematics-related affect: Embodied and social theories. *Research in Mathematics Education*, *14*(2), 137–161.

Hannula, M. S. (2019). Young learners' mathematics-related affect: A commentary on concepts, methods, and developmental trends. *Educational Studies in Mathematics*, *100*(3), 309–316. https://doi.org/10.1007/s10649-018-9865-9

Hannula, M. S. (2011). The structure and dynamics of affect in mathematical thinking and learning. Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education: Cerme 7, 9th - 13th February 2011, Rzeszów, Poland, 34–60. https://researchportal.helsinki.fi/en/publications/the-structure-and-dynamics-of-affect-in-mathematical-thinking-and

Hannula, M. S. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics 63*, 165–178. doi: 10.1007/s10649-005-9019-8

Lange, T. (2009) "Tell them that we like to decide for ourselves" – Children's agency in mathematics education. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.) *Proceedings of the sixth congress of the European Society for Research in Mathematics Education*, Lyon.Leontyev, A. N. (n.d.). *Activity and Consciousness*. 196.

Lewis, G. (2013). Emotion and disaffection with school mathematics. *Research in Mathematics Education*, *15*(1), 70–86. https://doi.org/10.1080/14794802.2012.756636

McGeown, S. P., & Warhurst, A. (2020). Sex differences in education: Exploring children's gender identity. *Educational Psychology*, *40*(1), 103–119. https://doi.org/10.1080/01443410.2019.1640349

Nielsen, H. B. (2017). Chapter 15: Gender as Analytic, Political and Interdisciplinary Concept. *Concepts in Action*, 264–283. https://doi.org/10.1163/9789004314207_016

Nyman, M. (In press). Primary student's expressed emotions towards mathematics.

Nyman, M., & Sumpter, L. (2019). The issue of 'proudliness': Primary students' motivation towards mathematics. *LUMAT*, *2*. https://doi.org/10.31129/LUMAT.7.2.331

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester's (Ed) Second handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics. Charlotte, NC: Information Age Pub.

OECD (2019), *PISA 2018 Results (Volume II): Where All Students Can Succeed*, PISA, OECD Publishing, Paris, https://doi.org/10.1787/b5fd1b8f-en.

Radford, L. (2015). Of love, frustration and mathematics: A cultural-historical approach to emotions in mathematics teaching and learning. In B. Pepin, B. Roesken-Winter (eds.), *From beliefs todynamic affect systems in mathematics education*, (pp. 25–49). doi: 10.1007/978-3-319-06808-4_2

Radford, L. (2018). Teaching and learning (algebra or something else): Working together to make sense of similarities and differences between theories (and understanding oneself). Paper Presented at the AERA Symposium Dealing with Diverse Discourses (3D): Can We Build on Each Other's Research Contributions?, New York.

Ryan, R. M., & Deci, E. L. (2000). Self-determination theory and the facilitation of intrinsic motivation, social development, and well-being. *American Psychologist*, *55*(1), 68.

Samuelsson, 2011

Schirmer, A. (2015). *Emotion*. SAGE Publications, Inc. http://sk.sagepub.com/books/emotion

Schukajlow, S., Rakoczy, K., & Pekrun, R. (2017). Emotions and motivation in mathematics education: Theoretical considerations and empirical contributions. *ZDM -SPRINGER-*, *3*, 307.

Schunk, D. H., Pintrich, P. R., & Meece, J. L. (2010). *Motivation in education: Theory, research, and applications* (3. ed., Internationl ed.). Pearson Education International.

Skolverket (2018, August) *Lista över socioekonomiskt index och den förväntade andelen obehöriga elever till gymnasieskolan.* https://www.skolverket.se/download/18.4fc05a3f164131a7418133b/1535448111495/Lista_socioekonomiskt_index-2019.pdf

Stoet, G., & Geary, D. C. (2018). The Gender-Equality Paradox in Science, Technology, Engineering, and Mathematics Education. *Psychological Science, 29(4)*, 581–593. https://doi.org/10.1177/0956797617741719

Suárez-Álvarez, J., Fernández-Alonso, R., & Muñiz, J. (2014). Self-concept, motivation, expectations, and socioeconomic level as predictors of academic performance in mathematics. *Learning & Individual Differences*, *30*, 118–123. https://doi.org/10.1016/j.lindif.2013.10.019

Sumpter, L. (2012). Upper secondary school students' gendered conceptions about affect in mathematics. *Nordic Studies in Mathematics Education*, 17 (2), 27–47.

Sutter-Brandenberger, C. C., Hagenauer, G., & Hascher, T. (2018). Students' self-determined motivation and negative emotions in mathematics in lower secondary education – Investigating reciprocal relations. *Contemporary Educational Psychology*, *55*, 166–175. https://doi.org/10.1016/j.cedpsych.2018.10.002

Takeuchi, M. A., Towers, J., & Plosz, J. (2016). Early Years Students' Relationships with Mathematics. *Alberta Journal of Educational Research*, *62*(2), 168–183.

Wedege, T. (2011). Doing gender in mathematics education. In G. Brandell & A. Pettersson (Eds.), *Matematikundervisning: Vetenskapliga perspektiv* 92–114. Stockholm: Stockholms universitets förlag.

Vetenskapsrådet. (2017). *God forskningssed* [Good research practice]. Vetenskapsrådets rapportserie 1:2001.

ISBN 978-91-7797-988-3

Department of Mathematics
and Science Education

Stockholms
universitet